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Résumé de l'article

La dynamique des baies de vêlage et le retrait de l'inlandsis le long du Saint-Laurent. Les courant de glace qui drainent l'extension de l'inlandsis en milieu marin peuvent être particulièrement sujets à des retraits catastrophiques puisqu'ils s'écoulent le long de chenaux surcreusés dans la roche en place et que la migration de la ligne d'ancrage risque de creuser une baie de vêlage remplie par une banquise ou des icebergs. Les données géologiques suggèrent qu'une baie de vêlage s'est formée dans le chenal laurentien et dans la vallée du Saint-Laurent après le maximum du Wisconsinien supérieur. On a calculé des vitesses de retrait dans cette baie selon une variété de modèles en supposant que, localement, l'inlandsis laurentidien s'est étendu jusqu'à la limite du plateau continental. Si une banquise se forme à l'aval de la ligne d'ancrage en retrait et que la tension de cisaillement entre la banquise et sa bordure est de 1 bar, on ne connaîtra qu'un retrait de 150 km. Tout retrait ultérieur exige une lubrification par une glace dont l'orientation cristallographique préférentielle est en fonction de la contrainte qui se crée entre la banquise et ses côtés adjacents ou une désintégration complète de la banquise. Il faut de 3000 à 6000 ans pour effectuer un retrait de 300 km. Par la suite, le retrait se poursuit rapidement jusqu'à ce qu'une nouvelle ligne d'ancrage en équilibre s'établisse à quelque 1 100 km de la bordure du plateau continental dû à la présence d'une banquise bien lubrifiée. Si un vêlage intensif d'icebergs se produit à la ligne d'ancrage ou à proximité, le retrait devrait se poursuivre le long du Saint-Laurent jusqu'au lac Ontario. Selon différents modèles, le temps minimal requis pour que s'effectue le retrait, à partir d'un point à 300 km à l'amont de la bordure du plateau continental jusqu'au lac Ontario, est d'environ 2000 ans.

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CALVING BAY DYNAMICS AND ICE SHEET RETREAT UP THE ST. LAWRENCE VALLEY SYSTEM

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ABSTRACT Ice streams that drain marine ice sheets are particularly susceptible to catastrophic retreat because they flow through bedrock troughs, and grounding line migration would produce a calving bay filled either with an ice shelf or with icebergs. Geological evidence suggests that a calving bay formed in the Laurentian Channel and the St. Lawrence valley after the late-Wisconsin maximum. Retreat rates in this calving bay are calculated for a variety of possible models assuming that locally the late-Wisconsin Laurentide ice sheet extended to the edge of the continental shelf. If an ice shelf forms in front of the retreating grounding line, and the shear stress between the ice shelf and its margins is one bar, retreat continues for only 150 km. Further retreat requires lubrication by ice with a strain-dependent preferred crystal fabric that develops between the ice shelf and its sides, or by complete removal of the ice shelf. Under these conditions the first 300 km of retreat takes at least 3000 to 6000 years. Thereafter, further retreat is rapid until, if a lubricated ice shelf is present, a new equilibrium grounding line is established about 1100 km from the edge of the continental shelf. If massive calving of icebergs occurred at, or near the grounding line, then retreat would continue up the St. Lawrence valley through to Lake Ontario. Of the various models considered, the minimum time taken for retreat from a point 300 km inland from the edge of the continental shelf through to Lake Ontario is about 2000 years.

RÉSUMÉ *La dynamique des baies de vêlage et le retrait de l'inlandsis le long du Saint-Laurent.* Les courant de glace qui drainent l'extension de l'inlandsis en milieu marin peuvent être particulièrement sujets à des retraits catastrophiques puisqu'ils s'écoulent le long de chenaux surcreusés dans la roche en place et que la migration de la ligne d'ancrage risque de creuser une baie de vêlage remplie par une banquise ou des icebergs. Les données géologiques suggèrent qu'une baie de vêlage s'est formée dans le chenal laurentien et dans la vallée du Saint-Laurent après le maximum du Wisconsinien supérieur. On a calculé des vitesses de retrait dans cette baie selon une variété de modèles en supposant que, localement, l'inlandsis laurentidien s'est étendu jusqu'à la limite du plateau continental. Si une banquise se forme à l'aval de la ligne d'ancrage en retrait et que la tension de cisaillement entre la banquise et sa bordure est de 1 bar, on ne connaîtra qu'un retrait de 150 km. Tout retrait ultérieur exige une lubrification par une glace dont l'orientation cristallographique préférentielle est en fonction de la contrainte qui se crée entre la banquise et ses côtés adjacents ou une désintégration complète de la banquise. Il faut de 3000 à 6000 ans pour effectuer un retrait de 300 km. Par la suite, le retrait se poursuit rapidement jusqu'à ce qu'une nouvelle ligne d'ancrage en équilibre s'établisse à quelque 1 100 km de la bordure du plateau continental dû à la présence d'une banquise bien lubrifiée. Si un vêlage intensif d'icebergs se produit à la ligne d'ancrage ou à proximité, le retrait devrait se poursuivre le long du Saint-Laurent jusqu'au lac Ontario. Selon différents modèles, le temps minimal requis pour que s'effectue le retrait, à partir d'un point à 300 km à l'amont de la bordure du plateau continental jusqu'au lac Ontario, est d'environ 2000 ans.

РЕЗЮМЕ ДИНАМИКА БУХТ. ОБЛАМЫВАЮЩИЕ *ЛЕДНИКИ*, И ОТСТУПАНИЕ *ЛЕДНИКОВЫХ* ПОКРОВОВ В ДОЛИНЕ РЕКИ СВ.ЛАВРЕНТИЯ. Из-за потоков в ледниках которые дренировали морские ледниковые покровы, потоки были особенно восприимчивы к катастрофическому отступанию потому что потоки текли по мульдам в коренной породе. Эти миграции базисных линий могли сформировать бухту в которой обламывался ледник и наполнял ее шельфовым льдом или айсбергами. Геологические данные наводят *на мысль о том что такая бухта образовалась* в лаврентийском русле и в долине Св. Лаврентия после поздне-висконсинского максимума. Скорости отступления в этой бухте калькулируются для множество возможных схем основанных на предположении что поздне-висконсинский Лаврентийский ледниковый покров локально простирался до края материкового шельфа. Если ледниковый покров образовывается впереди отступающей базисной линии и если *сдвигающее напряжение между шельфовым* ледником и его краями ровно одному бару, то отступание продолжается только на 150км Дальнейшее отступание требует или смазку льдом преимущественно зависимого от напряжения, кристаллической структурой которая образовывается между шельфовым ледником и его краями, или при полном смещении шельфового ледника. При таких условиях минимум от 3 000 до 6 000 лет понадобятся для отступания на первые 300 км. Дальнейшее отступание идет быстро до того как, если присутствует смазанный шельфовый ледник, установится новая уравновешенная базисная линия приблизительно в 1 100км от края материкового шельфа. Если образование многочисленных айсбергов происходило на или недалеко от базисной линии, то отступание продолжалось бы вверх по долине реки Св.Лаврентия до самого озера Онтарио. После рассмотрения разных схем было выдвинуто предположение, что понадобилось бы минимум 2 000 лет для того чтобы ледник отступил до озера Онтарио от того места, которое находилось в 300км от края материкового шельфа.

INTRODUCTION

A marine ice sheet is one that is grounded on rock that was below sea level before isostatic rebound. Near the edge of a marine ice sheet the ice becomes sufficiently thin to float free from bedrock and to form an ice shelf. Most of the West Antarctic ice sheet is grounded well below sea level and it forms the only major existing marine ice sheet. However, HUGHES *et al.* (in press) have reviewed evidence suggesting that large portions of the Pleistocene ice sheets in the northern hemisphere were marine ice sheets. WEERTMAN (1974), by solving the steady-state continuity equation at the grounding line, has shown that a two-dimensional marine ice sheet flowing into an unconfined ice shelf is inherently unstable: it either grows over a flat sea bed until it reaches the edge of the continental shelf or, once retreat begins, it shrinks irreversibly and finally disappears. Thomas (unpubl.) has extended this analysis to a growing or shrinking three-dimensional ice sheet flowing over a wavy bed into a confined ice shelf. Calving bays form where ice sheet collapse occurs over a bed-rock trough, so we shall briefly review the dynamics of a collapsing marine ice sheet. Readers who wish to omit this mathematical section may turn to page 350, where the results are applied to the Laurentian Channel.

THE MODEL

Consider a vertical section taken between two adjacent flow lines through an ice stream that drains a marine ice sheet, and across its fringing ice shelf (Fig. 1). At the grounding line between ice sheet and ice shelf the section is vertical and of unit width. The *x*-axis is taken at sea level in the direction of movement, with the origin beneath the ice sheet summit, the position of which is assumed to be time-invariant. The *z*-axis is vertical and positive upwards. The effect of the third dimension is incorporated by allowing the width of the section to vary with *x* and *z* in conformity with convergent and divergent flow.

Since the ice sheet is not in steady state the section may grow or shrink in the z direction with time. For a shrinking ice sheet the terminology is:

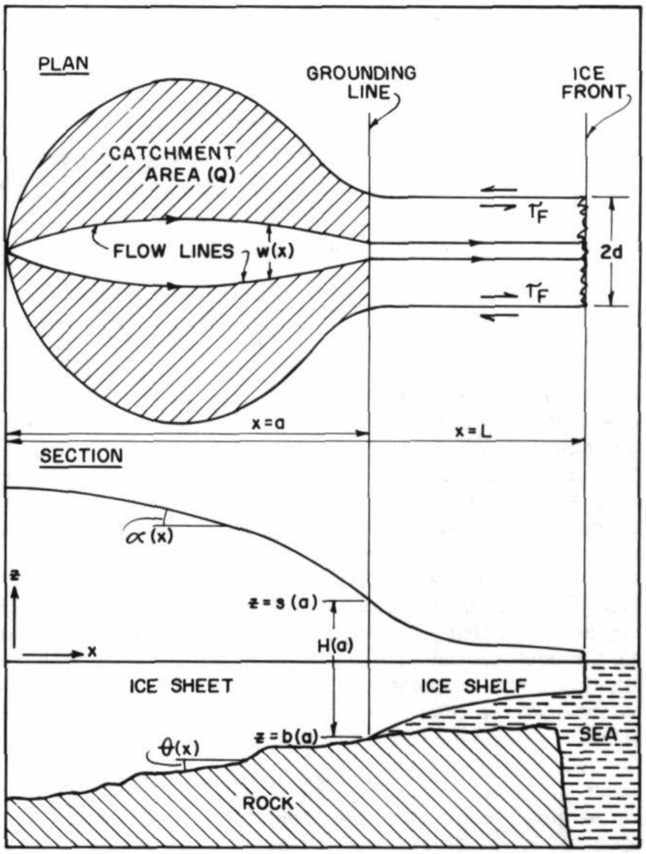

FIGURE 1. Part of a marine ice sheet. The catchment area of an ice stream flowing into a bounded ice shelf is shown together with the coordinate system that is used in the text. In this example, $\theta(x)$ is positive and $\alpha(x)$ is negative.

Une partie de l'extension d'un inlandsis en milieu marin. On y montre la surface de réception d'un courant de glace qui s'écoule vers une banquise ainsi que le système de coordonnées utilisé. Dans ce cas, $\theta(x)$ est positif tandis que $\alpha(x)$ est négatif.

a	The value of x at the grounding line.
$\dot{a}$	The rate of grounding line advance.
$H(x)$	Ice thickness at x
$b(x)$	Value of z at the base of the ice sheet at x.
$s(x)$	Value of z at the surface of the ice sheet at x.
$w(x, z)$	Width of the ice sheet section in the y direction at x and z.
$V(x, z)$	Ice velocity in the x direction at x and z.
$\theta(x)$	Bedrock slope at x.
$\alpha(x)$	Ice surface slope at x. All slopes are positive uphill and negative downhill.
$\dot{A}(x)$	Surface accumulation rate at x.
$\dot{F}(x)$	Basal freezing rate at x.
$\dot{H}(x)$	Rate of thickening of the ice sheet with time. $\dot{A}$, $\dot{F}$ and $\dot{H}$ are expressed as ice thickness per unit time.
$\dot{\epsilon}_{zz}(x, z)$	Vertical strain rate at x and z.

All of the above parameters are functions of time *(t)*.

ρ_i	Density of ice (assumed constant at 917 kg m $^{-3}$).
ρ_w	Density of sea water (assumed constant at 1030 kg m $^{-3}$).
g	Acceleration due to gravity (9.8 m s $^{-2}$).

The ice flow law:

$$\dot{\epsilon}_{ij} = \frac{\tau^{n-1}}{B^n} \sigma'_{ij}$$

where

$$i, j = x, y, z \ (i = j\text{: direct}; \ i \neq j\text{: shear}),$$

$$\dot{\epsilon}_{ij} = 1/2 \left[\frac{\partial U_{ij}}{\partial i} + \frac{\partial U_{ij}}{\partial j}\right]; \ U = \text{velocity},$$

$$\sigma'_{ij} \text{ (deviator stress)} = \sigma_{ij} - 1/3\ \delta\ \Sigma\sigma_{ij};$$

$$\sigma = \text{stress}; \ \delta = 1 \text{ for } i = j, \ \delta = 0 \text{ for } i \neq j.$$

$$\tau \text{ (effective shear stress)} = + [\Sigma\sigma'^{\,2}_{ij})/2]^{1/2},$$

n is a constant equal approximately to 3 (here $n = 3$ is assumed); B is a constant that depends on temperature and ice fabric.

From conservation of volume, continuity equations will be derived that relate the parameters defined above. Using expressions for the ice velocity and strain rate in terms of ice sheet geometry, the continuity equations will be solved for the rate of ice sheet thinning and for the rate of grounding line retreat.

Balancing upstream accumulation against ice flow and ice sheet thickening:

$$\int_0^x (\dot{A}(x) \cdot w(x, s,) - \dot{H}(x) \cdot \bar{w}(x) + \dot{F}(x) \cdot w(x, b))\, dx$$
$$= \int_{b(x)}^{s(x)} V(x, z) \cdot w(x, z)\, dz \qquad (1)$$

where $\bar{w}(x)$ is the value of w averaged over depth.

Differentiating with respect to x gives:

$$\dot{A}(x) \cdot \bar{w}(x, s) - \dot{H}(x) \cdot \bar{w}(x) + \dot{F}(x) \cdot w(x, b) =$$
$$\int_{b(x)}^{s(x)} (w(x,z) \frac{\partial V(x,z)}{\partial x} + V(x,z) \frac{\partial w(x,z)}{\partial x})\, dz +$$
$$V(x,s) \cdot w(x,s) \frac{\partial s}{\partial x} - V(x,b) \cdot w(x,b) \frac{\partial b}{\partial x}$$

And since

$$\frac{\partial V(x,z)}{\partial x} = \dot{\epsilon}_{xx}(x,z), \qquad \frac{V(x,z)}{w(x,z)} \cdot \frac{\partial w(x,z)}{\partial x} = \dot{\epsilon}_{yy}(x,z)$$

$$\frac{\partial s}{\partial x} = \alpha(x), \qquad \frac{\partial b}{\partial x} = \theta(x)$$

and, for incompressible ice,

$$\dot{\epsilon}_{xx} + \dot{\epsilon}_{yy} = -\dot{\epsilon}_{zz}$$

we can write:

$$\dot{A}(x) \cdot w(x,s) - \dot{H}(x) \cdot \bar{w}(x) + \dot{F}(x) \cdot w(x,b) = V(x,s) \cdot w(x,s) \cdot \alpha(x)$$
$$- V(x,b) \cdot w(x,b) \cdot \theta(x) - \int_{b(x)}^{s(x)} w(x,z) \cdot \dot{\epsilon}_{zz}(x,z)\, dz \qquad (2)$$

This is the equation of mass continuity for the case of constant density and it can be applied to an ice sheet if we express $\dot{A}$, $\dot{T}$ and $\dot{F}$ as ice thickness per unit time.

At the grounding line between a retreating marine ice sheet and its fringing ice shelf equation (2) can be simplified considerably. The ice velocity is probably high and movement is mainly by bottom sliding so that V is almost independent of z. Moreover, because the ice becomes afloat horizontal strain rates are also constant with depth. Finally w is, by definition, independent of z at the grounding line. So, writing

$$\dot{\epsilon}_{zz}(a,z) = \dot{\epsilon}_{zz}(a)$$
$$V(a,z) = V(a,s) = V(a)$$

and
$$w(a,z) = w(a)$$

equation (2) becomes:

$$\dot{A}(a) + \dot{F}(a) - \dot{H}(a) \sim V(a,s) \cdot [\alpha(a) - \theta(a)] - H(a) \cdot \dot{\epsilon}_{zz}(a) \qquad (3)$$

To proceed further it is necessary to express V and $\dot{\epsilon}_{zz}$ in terms of H and α. WEERTMAN (1974) has suggested that, near the grounding line, the values of V and $\dot{\epsilon}_{zz}$ are both influenced by the grounded ice slope-induced stresses and by the ice shelf spreading stresses. However, for simplicity, we shall adopt a sliding velocity law of the type proposed by WEERTMAN (1959) and KAMB (1970):

$$V(a) = C\left|\rho_i\, gH(a) \cdot \alpha(a)\right|^m \qquad (4)$$

where $m \sim 2$ and C is a constant determined mainly by the bed roughness.

The vertical strain rate in a confined ice shelf is given by THOMAS (1973):

$$\dot{\epsilon}_{zz} = ([\frac{\rho_i\, gs(a)}{2} - \sigma_R]/\xi\, \bar{B})^3 \qquad (5)$$

where σ_R is the "back pressure" applied to the ice shelf by its margins and by obstructions such as ice rises; ξ is a factor determined by the ratios of the various strain rate components:

$\xi \sim -2$ for zero strain in the y direction

$\xi \sim -5/3$ for equal spreading in the x and y directions.

Here we shall assume that the ice shelf is parallel sided (Fig. 1) so that there is zero strain in the y direction and $\xi = -2$. $\bar{B}$ is the value of B averaged over depth.

For an actively retreating marine ice sheet the fringing ice shelf is probably free of major ice rises; a large ice rise would increase the magnitude of σ_R sufficiently to delay retreat (THOMAS, 1976). Thus σ_R is determined by friction between the ice shelf and its margins and, from THOMAS (1973):

$$\sigma_R = \frac{\tau_F}{H(a)} \int_a^L \frac{H(x)}{d}\, dx \qquad (6)$$

where τ_F is the average shear stress between the ice shelf and its margins, L is the value of x at the front of the ice shelf and d is the half width of the ice shelf (Fig. 1). τ_F can be regarded as the equivalent of a plastic yield stress for ice; for polycrystalline ice, $\tau_F \sim$ one bar (10^5 N m^{-2}). However, observations on existing fast glaciers (HUGHES, 1975) suggest that their floating tongues are lubricated by a band of anisotropic ice that forms along each margin in response to the intense shear in these regions. This has the effect of apparently reducing τ_F to about 0.3 bars on the floating tongue of the Byrd Glacier in Antarctica moving at 800 m a^{-1} (metres per year) and probably less on the Jacobshavn Glacier in Greenland moving at 8 km a^{-1}.

For the present case equation (6) becomes:

$$\sigma_R \sim \beta \tau_F (L - a)/d \qquad (7)$$

where

$$\beta = \frac{1}{H(a) \cdot [L - a]} \int_a^L H(x)\, dx \sim 0.5 \rightarrow 1$$

Assuming that $\dot{F} << \dot{A}$ equation (1) can be written as:

$$V(a) \sim \phi\, [\bar{\dot{A}} - \bar{\dot{H}}]\, a / H(a) \qquad (8)$$

where $\bar{\dot{A}}$ and $\bar{\dot{H}}$ are values of $\dot{A}$ and $\dot{H}$ averaged between $x = 0$ and $x = a$, ϕ is a factor that corrects for the effects of diverging and converging flow lines: $\phi \sim Q/(2ad)$ where Q is the area of the catchment basin (Fig. 1). During rapid retreat of the grounding line a marine ice sheet thins most rapidly at the edge and least rapidly at the centre and, assuming a linear increase in $|\dot{H}(x)|$ from zero at the summit to $|\dot{H}(a)|$ at the grounding line, $\bar{\dot{H}} = \frac{1}{2} \dot{H}(a)$.

Equations (3), (4), (5), (7) and (8) can now be solved to give the thinning rate at the grounding line $[-\dot{H}(a)]$ in terms of snow accumulation rate $\dot{A}$, basal slope θ, ice thickness and surface elevation at the grounding line $H(a)$ $[= - b(a) \cdot \rho_w / \rho_i]$ and $s(a)$ $[= (1 - \frac{\rho_w}{\rho_i})\, b(a)]$, the sliding law constants C and m, the flow law constant B, the shear stress τ_F between the ice shelf and its margins, and the dimensions of the catchment basin and of the ice shelf. Finally the retreat rate of the grounding line $[-\dot{a}]$ can be calculated from:

$$\dot{a} = \frac{\dot{H}(a)}{[1 - \rho_w/\rho_i]\,\theta(a) - \alpha(a)} \qquad (9)$$

For an existing ice sheet, values can be assigned to the unknown parameters from the results of field measurements, but currently there is considerable uncertainty in the values of C, m and τ_F. Here, we shall try to simulate the behaviour of a calving bay in the Laurentian Channel and it is necessary to derive values for the relevant parameters from current topography, Laurentide ice sheet reconstruction, and analogy with present-day ice sheets.

THE ST. LAWRENCE CALVING BAY

Theoretical reconstruction of the 18,000 yrs. BP Laurentide ice sheet (by the University of Maine at Orono (UMO) for CLIMAP) suggests that the Laurentian Channel was the site of a major glacier drainage basin. Since much of the ice stream that occupied this basin was grounded well below sea level, we may use the equations from the previous section to examine its stability and to calculate possible retreat rates. The UMO ice sheet reconstruction can be used to estimate topography beneath the Laurentide ice sheet if we assume that: i) because of lithosphere rigidity the Earth's crust achieves regional isostatic equilibrium with the ice load within 250 km radius (BROTCHIE and SILVESTER, 1969); ii) the effective eustatic sea level was 120 m lower than today; and iii) the present topography is in isostatic equilibrium.

Figure 2 shows the 400 m bathymetric contour for 18,000 BP in the St. Lawrence area, and a depth profile taken along the centre of the Laurentian Channel and up the St. Lawrence River. For solution of the equations in the previous section we require depth during retreat, when the land was actively rising. However, for a sufficiently rapid retreat, the uplift rate would be considerably less than the ice thinning rate, which may have reached several m a^{-1}. Moreover, the profile in Figure 2 was calculated from present-day bathymetric contours. Deposition since retreat of the ice has probably buried the 18,000 BP surface, which would thus be deeper than shown in Figure 2. This error is of opposite sign to errors introduced by uplift and, since little is known about uplift rates during retreat, the errors are assumed to cancel so that values of $b(a)$ and $\theta(a)$ can be taken from the calculated profile. The dimensions of the catchment basin are estimated from the UMO reconstruction of the Laurentide ice sheet as: $Q \sim 4 \times 10^5$ km^2; L $\sim$ 2000 km. Snow accumulation rates and surface temperatures are taken from SUGDEN (1977):

$\dot{A}(L) = 0.7$ m $a^{-1} \rightarrow \dot{A}(L\text{-}800$ km$) = 0.15$ m a^{-1},

$\bar{\dot{A}} = 0.25$ m a^{-1} at $a = L \rightarrow \bar{\dot{A}} = 0.1$ m a^{-1} at $a = L - 800$ km.

The ice flow law parameter $\bar{B}$ is a function of ice temperature, and near the ice margin the surface tem-

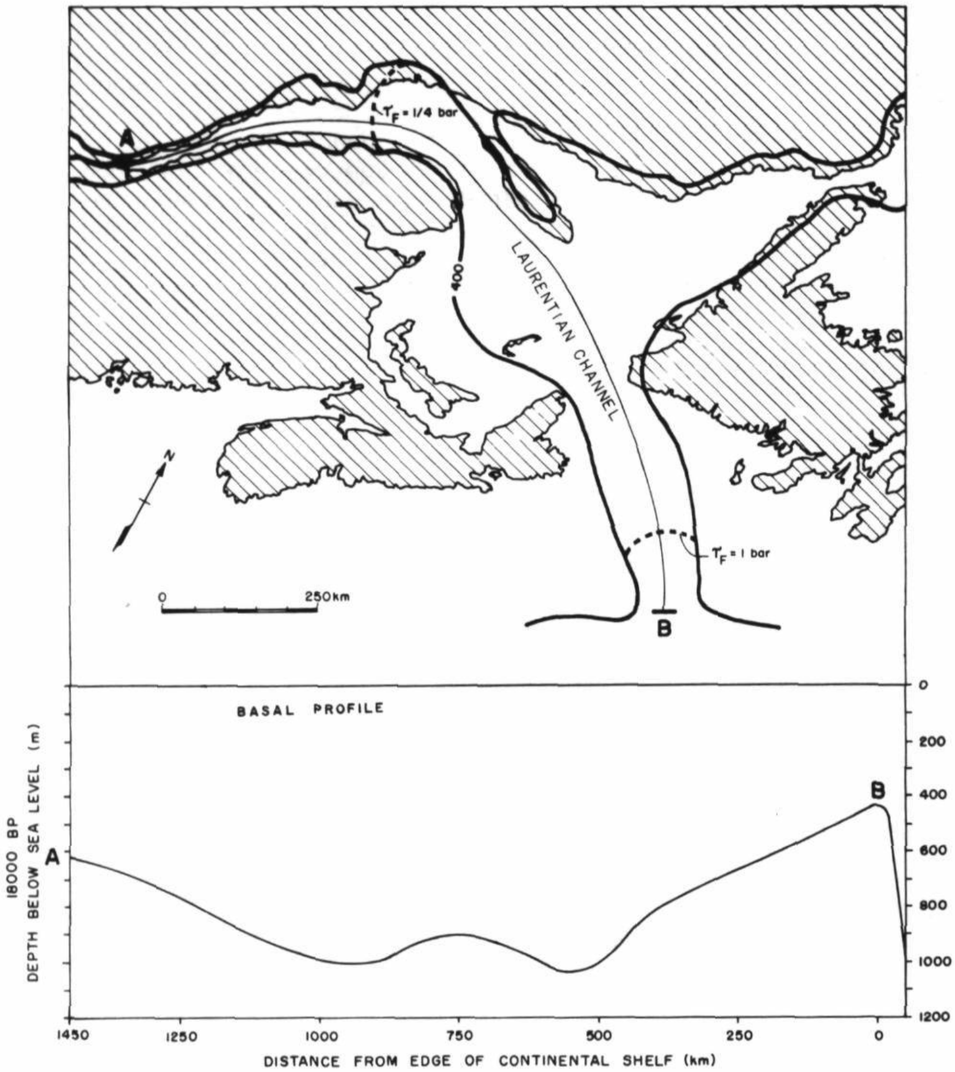

FIGURE 2. The St. Lawrence seaway and the Laurentian Channel with the 400 m bathymetric contour for 18,000 BP. The late-Wisconsin Laurentide ice sheet is assumed to have extended to B, the edge of the continental shelf. The basal profile AB is used in the text to calculate retreat rates of the ice sheet/ice shelf grounding line. The dashed lines mark stable positions of the grounding line if an ice shelf forms to seaward, for two assumed values of the shear stress between the ice shelf and its margins (τ_F = 1 bar and τ_F = 1/4 bar).

L'isobathe de 400 m vers 18 000 ans BP dans la voie maritime du Saint-Laurent et le chenal laurentien. On présume que l'inlandsis laurentidien du Wisconsinien supérieur s'est étendu jusqu'au point B, à la limite du plateau continental. On utilise le profil de la base AB pour calculer la vitesse du retrait de la ligne d'ancrage de l'inlandsis et de la banquise. Les lignes brisées indiquent les positions fixes de la ligne d'ancrage lorsqu'une banquise se forme au large pour deux valeurs présumées de la tension de cisaillement entre la banquise et sa bordure (τ_F = 1 bar et τ_F = 1/4 bar).

perature $\sim -10°C$ giving (from THOMAS, 1973) $\overline{B} \sim 3.2 \times 10^5$ N m^{-2}a$^{1/3}$ assuming that n = 3; 200 km inland the surface temperature drops to $-20°C$ ($\overline{B} \sim 4.7 \times 10^5$ N m^{-2} a$^{1/3}$). Further inland $\overline{B} = 4.7 \times 10^5$ N m^{-2} a$^{1/3}$ is assumed.

The Laurentian Channel forms a trough that is between 80 km and 100 km wide and if rapid ungrouding of the ice sheet occured in this area it was probably, in its early stages, confined to this trough. Thus the half-width at the grounding line is assumed to be $d \sim 40$ km. The value of ϕ can be calculated from d and the dimensions of the catchment basin to give $\phi \sim 2.5$. This is assumed to remain constant during retreat. If ice shelf forms downstream from the grounding line it is assumed to be parallel sided and to extend to the edge of the continental shelf (B in Fig. 2). The value of β is assumed to be 0.75.

In order to examine the influence of the sliding law parameter C and the shear stress τ_F between the ice shelf and its margins, six models will be considered:

(1) $C = 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 10^5$ N m^{-2} (one bar)
(2) $C = 2 \times 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 10^5$ N m^{-2}
(3) $C = 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 0.25 \times 10^5$ N m^{-2}
(4) $C = 2 \times 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 0.25 \times 10^5$ N m^{-2}
(5) $C = 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 0$
(6) $C = 2 \times 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$;
$\tau_F = 0$.

The sliding law parameter m is assumed to equal 2 and the values of C are calculated using velocity, ice thickness and surface slope data from ice stream "B" in the West Antarctic ice sheet (THOMAS, 1976) and from the Jacobshavn Glacier in Greenland (unpublished data collected by the U.S. Coastguard Jacobshavn Glacier survey, 1976), which give values of $C \sim 2 \times 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$ and $C \sim 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$ respectively. Models (1) and (2) assume that the ice shelf downstream of the grounding line has no preferred ice fabric, models (3) and (4) allow the development of "soft" ice with a highly developed fabric to form along the lateral margins of the ice shelf and models (5) and (6) assume that the ice calves at the grounding line to form icebergs rather than an ice shelf.

RESULTS

Equations (3), (4), (5), (7) and (8) can be combined to give:

$$H(a) \sim \frac{\dot{A}(a) + \dfrac{\phi \bar{\dot{A}}\, a}{H(a)}\left(\left[\dfrac{V(a)}{C}\right]^{1/2} \dfrac{1}{\rho_i g H(a)} + \theta(a)\right) - H(a)\left\{\dfrac{\rho_i g s(a)}{4\bar{\bar{B}}} - \dfrac{\beta \tau_F (L-a)}{2d\,\bar{\bar{B}}}\right\}^3}{1 + \dfrac{\phi a}{2H(a)}\left(\left(\dfrac{V(a)}{C}\right)^{1/2} \dfrac{1}{\rho_i g H(a)} + \theta(a)\right)} \qquad (10)$$

where $V(a) \sim \phi a\, [\bar{\dot{A}} - 0.5\, \dot{H}(a)]/H(a)$ (11)

Equation (10) can be solved iteratively for $\dot{H}(a)$, but first we shall consider the equilibrium situation when $\dot{H}(a) = 0$, *i.e.* when the ice sheet is neither growing nor shrinking. With $\dot{H}(a) = 0$ and $a = L$ equation (10) gives the value of ice thickness at the grounding line of an equilibrium ice sheet that extends to the edge of the continental shelf as:

$\overset{*}{H} \sim 475$ m for $C = 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$

and

$\overset{*}{H} \sim 450$ m for $C = 2 \times 10^{-6}$ m a^{-1} (N m^{-2})$^{-2}$

Thus the grounding line thickness is approximately 460 m when the ice sheet has reached its maximum size. Note that the term involving τ_F is zero. This means that a freely floating ice shelf extending beyond the continental shelf has no influence on ice sheet thickness or stability.

So long as sea-bed depth at the edge of the continental shelf is less than $\overset{*}{H}\rho_i/\rho_w$ conditions are favorable for ice sheet growth, assuming of course that snow accumulation is sufficient to nourish that growth. Since a growing ice sheet encounters a more or less undepressed sea-bed, growth continues until, in this case, the grounding line reaches the undepressed 400 m depth contour (with respect to prevailing sea level). At this equilibrium position the ice sheet continues to thicken until drainage balances snow accumulation. At the same time the sea bed at the margin is slowly depressed by the growing load of ice until the grounding line depth exceeds $\overset{*}{H}\rho_i/\rho_w$ when the grounding line starts to retreat. If there is a sufficiently high bedrock still upstream of the grounding line then the ice sheet shrinks to a new equilibrium size. Without such a sill, however, retreat is irreversible unless an embayed ice shelf forms to seaward of the grounding line and transmits a sufficiently large back-pressure due to shear between the ice shelf and its margins (this is the τ_F term in equation (10)) or to the pinning effects of grounded ice rises within the ice shelf. The constraining effects of ice rises are very large and may be sufficient to rapidly halt a grounding line retreat. The Laurentian Channel appears to be free from sea-bed high spots that would ground an ice shelf, and ice rise constraints are not included in equation (10).

The new equilibrium position of the grounding line that is imposed by ice shelf restraint can be found by solving equation (10) for a with $\dot{H}(a) = 0$, and using the assumed variation with a of $\dot{A}$, b, θ, and $\bar{B}$. For models (1) and (2), with τ_F = one bar, the new equilibrium position for the grounding line is within 100 to 150 km of the edge of the continental shelf (Fig. 2). This is a minor retreat and would scarcely affect the dimensions of the ice sheet. For models (3) and (4), with $\tau_F = 1/4$ bar, total retreat is between 1100 and 1150 km (Fig. 2). Finally, for icebergs calving at the grounding line ($\tau_F = 0$) models (5) and (6) give a total retreat that is more than 1800 km (the total length of the sampled profile). There is a considerable body of evidence indicating rapid retreat up the St. Lawrence prior to 13,000 BP (see other papers in this volume). This would suggest that retreat was facilitated either by the development of a lubricating band of soft ice in the zones of intense shear on each side of an ice shelf that formed in the wake of the retreating grounding line, by calving at the grounding line and rapid transport to seaward of the resulting icebergs so that no back-pressure was transmitted to the retreating ice sheet or, most probably, by a combination of these effects. Retreat along the valley currently occupied by the St. Lawrence River must have been accompanied by rapid calving of icebergs at or near the grounding line.

Solutions of equation (10) at 50 km intervals along the St. Lawrence depth profile (Fig. 2) give $\dot{H}(a)$, the thinning rate at the retreating grounding line, as a function of distance from the edge of the continental shelf. The ice velocity at the grounding line $V(a)$ can then be calculated using equation (11). The results are shown in Figure 3A. For the assumed depth profile (Fig. 2) the velocity of the ice stream when retreat began was $4 \rightarrow 5$ km a^{-1}. Models (1) and (2) show decreasing values of $V(a)$ as the grounding line retreated until, at $(L - a) \sim 150$ km, $V(a)$ drops to the equilibrium value (~ 1.5 km a^{-1}) necessary to drain upstream accumulation. Models (3) to (6) give maximum values of $V(a)$ at $(L - a) \sim 570$ km that lie between 10 km a^{-1} and 35 km a^{-1}. Although these ice velocities are very high they are not unreasonably so because the Jacobshavn Glacier, with an active front that is only about 6 km wide, has currently recorded ice velocities in excess of 8 km a^{-1}.

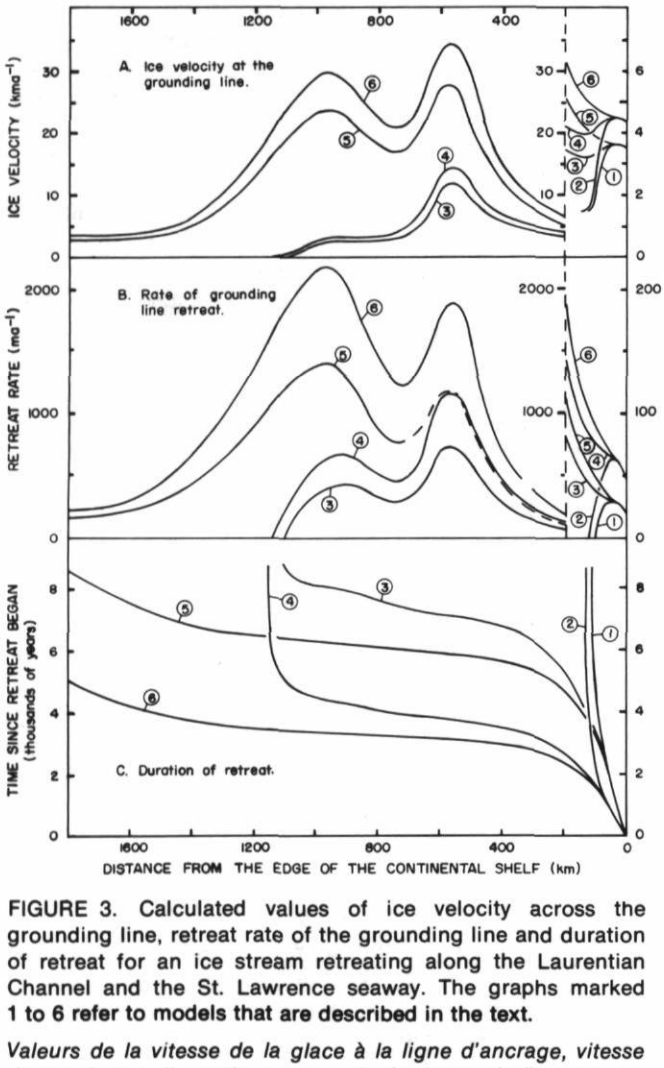

FIGURE 3. Calculated values of ice velocity across the grounding line, retreat rate of the grounding line and duration of retreat for an ice stream retreating along the Laurentian Channel and the St. Lawrence seaway. The graphs marked 1 to 6 refer to models that are described in the text.

Valeurs de la vitesse de la glace à la ligne d'ancrage, vitesse du retrait de la ligne d'ancrage et durée du retrait d'un courant de glace en retrait le long du chenal laurentien et de la voie maritime du Saint-Laurent. Les courbes 1 à 6 réfèrent aux modèles décrits dans le texte.

The rate of grounding line retreat $-\dot{a}$ is calculated from $\dot{H}(a)$ using equation (9) and the results are included in Figure 3B. For each of the models the retreat rate was slow (< 100 m a^{-1}) within 100 km of the edge of the continental shelf. Further inland, models (3) to (6) give retreat rates that initially increase very rapidly to more than 500 m a^{-1}, then fluctuate in accord with bedrock topography and finally decrease. Models (5) and (6) do not achieve equilibrium ($\dot{a} = 0$) and, at a distance of 1800 km from the edge of the continental shelf the grounding line continues to retreat at between 100 and 200 m a^{-1}.

The values of $\dot{a}$ can be used to reconstruct a time scale for grounding line retreat and Figure 3C includes plots of grounding line position against time since retreat began. The initial very slow retreat rates are determined by the assumed depth at the edge of the continental shelf, $b(L)$. Here $|b(L)|$ is assumed to be 450 m, or 50 m greater than the equilibrium depth for zero retreat ($\overset{*}{b} \sim 400$ m). The rate at which $|b(L)|$ increased was probably controlled by isostatic depression of the sea bed and by eustatic changes in sea level. It should be noted, however, that $\overset{*}{b}$ could be exceeded without any increase in $|b(L)|$, since $\overset{*}{b}$ is determined by the size of the catchment area, snow accumulation rates, ice temperature and glacier sliding law parameters. Appropriate changes in any of these variables could reduce $\overset{*}{b}$ and initiate retreat. Thus the time scale for the early stages of retreat in Figure 3C is somewhat arbitrary. However, after a total retreat of about 200 km the retreat rates are almost entirely determined by prevailing bedrock topography and by the magnitude of τ_F. Thus Figure 3C probably gives a fairly reliable estimate of time taken for grounding line migration between $(L - a) \sim 200$ km and $(L - a) \sim 1400$ km.

At $(L - a) = 1450$ km the retreating ice reached the site of Québec City and the valley of the present St. Lawrence River. In calculating the terminal values of $\dot{a}$ corresponding to $(L - a) > 1450$ km for models (5) and (6) the parameters ϕ and a were increased respectively to 4 and 450 km in an attempt to take account of the probable increase in catchment area associated with the formation of a calving bay. However, $\dot{a}$ is most sensitive to changes in the values assigned to the bottom topography. Figure 4 is a plot of $\dot{a}$, the grounding line retreat rate for models (5) and (6), against $-b(a)$, the depth below sea level of the bedrock, assuming a flat sea bed ($\theta = 0$). The effect of an increasing catchment area is included by setting $\phi = 3$ for one set of curves and $\phi = 6$ for a second set. The value of a is assumed to be 450 km and the other parameters take the values previously assumed for $(L - a) > 800$ km.

The St. Lawrence River flows through a broad valley with a surface elevation that is currently less than 100 m above sea level, and width that increases from about

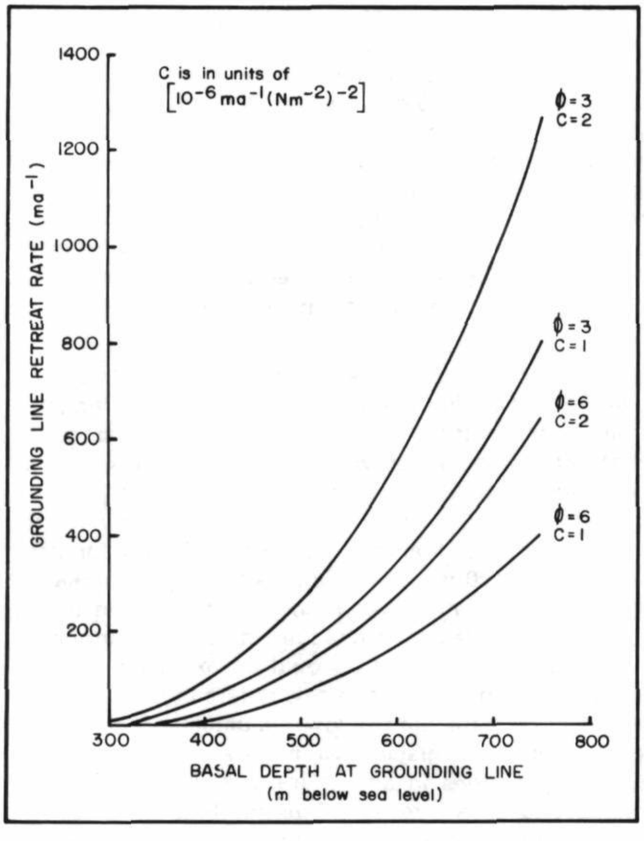

FIGURE 4. The rate of grounding line retreat through the St. Lawrence lowlands for different values of the sliding law constant and for different catchment areas.

Vitesse de retrait de la ligne d'ancrage dans les basses terres du Saint-Laurent pour différentes valeurs de la loi de la constance du glissement et pour différentes surfaces de réception.

30 km at Québec City to about 100 km further south. The total distance from Québec City at one end of this valley to Lake Ontario at the other is about 500 km. The UMO reconstruction of the 18,000 BP Laurentide ice sheet gives an ice surface elevation in this area of about 2,300 m. Thus, maximum isostatic depression was about 800 m lower than today, assuming isostatic balance with the regional (within 250 km) ice load. During ice retreat the eustatic sea level was about 120 m lower than today, giving St. Lawrence River valley depths below prevailing sea level between 580 m at the valley walls and 700 m or more in the present river basin. The shallower of these depths corresponds to the broad valley below the present 100 m a.s.l. contour. For a glacier calving across this valley ϕ takes a smaller value than for a glacier calving across an ice front of equal width to the present St. Lawrence River. Thus an indication of maximum probable retreat rates can be estimated from Figure 4 if we assume that the $\phi = 3$ curves at sea depth = 580 m represent flow along the entire St. Lawrence River valley and the $\phi = 6$ curves at sea depth = 700 m represent flow along the present bed of the St. Lawrence River. Both sets of curves give retreat rates of 300 to 500 m a^{-1} depending on the value of the sliding law parameter C. Thus complete retreat from Québec City through to Lake Ontario could have occured within a period of 1000 to 1700 years.

CONCLUSIONS

At the grounding line of an ice sheet there is a rapid transition from grounded ice sheet dynamics to ice shelf dynamics. In addition, for a marine ice sheet, the bedrock slopes downward towards the centre of the ice sheet. During growth, such an ice sheet continues to spread until it reaches the edge of the continental shelf, when it thickens to an equilibrium edge thickness with upstream snow accumulation exactly balanced by outflow across the grounding line. Thereafter retreat of the ice sheet can be initiated by: (i) reduction in equilibrium edge thickness following appropriate changes in upstream snow accumulation rate, catchment area, basal sliding conditions or ice temperature; (ii) isostatic depression of the sea bed as a delayed response to loading by the ice sheet; (iii) rising sea level near the margins of the ice sheet caused by either eustatic changes or by lateral gravitational drag by the ice sheet (CLARK, 1976).

Under suitable conditions the retreat can be irreversible unless sufficient back pressure is transmitted to the ice sheet by a floating ice shelf that forms to seaward of the retreating grounding line. Drainage from marine ice sheets is mainly through fast-moving ice streams that are particularly susceptible to catastrophic retreat because they flow through bedrock troughs. Grounding-line migration up one of these would produce a calving bay filled either with an ice shelf or with an array of icebergs. Several calving bays formed at different stages during the retreat of the Laurentide ice sheet and one of the earliest to form was probably in the Laurentian Channel, where the equilibrium edge thickness is calculated to be about 460 m corresponding to a sea depth of about 400 m. This depth may have been exceeded sometime during the growth of the ice sheet because the sea bed beneath the edge of a reconstructed Laurentide ice sheet was probably depressed to at least 450 m below the prevailing sea level. Grounding line retreat would begin as soon as the equilibrium edge thickness was exceeded, but it would take place very slowly and the sea bed would continue to isostatically sink. In this paper retreat rates have been calculated for an ice sheet retreating through the Laurentian Channel and up the St. Lawrence River assuming a sea bed topography appropriate to the 18,000 BP Laurentide ice sheet. This is a somewhat arbitrary assumption and the initial, rather slow retreat rates are very sensitive to the assumed bed topography.

If an ice shelf forms in front of the retreating grounding line and the shear stress between the ice shelf and its sides is one bar, retreat stops within about 150 km of the edge of the continental shelf. Further retreat requires lubrication by ice with a strain dependent preferred crystal fabric that develops between the ice shelf and its sides, or alternatively, by complete removal of the ice shelf. Under these conditions the first 300 km of retreat takes about 3000 to 6000 years. This estimate does not take account of the time taken for the sea bed at the edge of the ice sheet to be depressed the assumed 50 m beyond the depth for an equilibrium ice sheet. Since this depression probably is rather slow the total time taken for the grounding line to retreat the first 300 km is considerably greater than 3000 to 6000 years. Thereafter further retreat would be rapid until, if an ice shelf were present, a new equilibrium grounding line would be established about 1100 km from the edge of the continental shelf. The actual total retreat would depend mainly on the magnitude of the shear stress between the lubricated ice shelf and its sides (assumed here to be 1/4 bar).

If massive calving of icebergs occurred at, or near to the grounding line, then retreat would continue up the St. Lawrence valley through to Lake Ontario. Of the various models considered in this paper the minimum time taken for retreat from a point 300 km inland of the edge of the continental shelf through to Lake Ontario is about 2000 years. Isostatic uprise of the ice sheet bed and ice shelf formation during retreat would slow migration of the grounding line, so this estimate should be regarded as a minimum.

There is evidence for marine incursion as far inland as Ottawa by 12,800 BP (GADD, 1975). If this was caused by sea water flowing up the St. Lawrence valley then the first 300 km of ice sheet retreat must have been complete before, and probably well before, 15,000 BP. Since the first 300 km of retreat may have taken at least 3000 to 6000 years it probably began before and perhaps several thousand years before 18,000 BP. This places the timing of initial retreat up the Laurentian Channel before the late-Wisconsin maximum, suggesting that the retreat was triggered by some combination of sea bed depression, locally rising sea level due to lateral gravitational attraction by the ice sheet, reduction in total upstream snow accumulation, or reduction in the bed friction of the ice stream that flowed through the Laurentian Channel.

During the very rapid retreat of the grounding line up the Laurentian Channel ice velocities were extremely high. Under these conditions a matrix of icebergs that formed at the grounding line would tend to jam together and ultimately to form an ice shelf. Thus, there was probably an ice shelf right to the edge of and possibly beyond, the continental shelf until the grounding line had retreated at least 700 km, when ice velocities began to decrease. Thereafter the ice shelf may have melted back rather rapidly during a period of climatic warming and there is evidence for warming before 14,000 BP (MÖRNER and DREIMANIS, 1973). With most of the ice shelf gone, retreat would accelerate allowing the grounding line to migrate along the St. Lawrence valley to Lake Ontario within 1000 to 2000 years. This phase of the retreat required regular removal of icebergs formed at or near the grounding line, where ice velocities reach 5 km a^{-1}. If severe jamming occurred in the valley bottleneck near Québec City then retreat would have been delayed.

Bottom cores taken along the centre of the Laurentian Channel and the St. Lawrence River may provide a test of the ideas presented in this paper. If marine organisms do not exist beneath an ice shelf then the earliest post-Wisconsin dates from the cores should correspond to the seaward edge (or ice front) of the ice shelf, and they may be underlain by a thick deposit of glacial mud that was deposited beneath the ice shelf by silt-laden water flowing from beneath the rapidly moving ice stream. Thus, we might expect a rapid migration of marine organisms up the Laurentian Channel prior to 14,000 BP, followed by a fairly steady migration at about 400 m a^{-1} along the St. Lawrence Valley. If marine organisms can exist beneath an ice shelf then their first appearance corresponds to the position of the grounding line and would probably be overlain by a thick deposit of glacial mud. The rate of migration of the marine organisms would then give a direct measure of grounding line retreat.

Portions of the ice sheet that were grounded on land that was above the 400 m depth contour (Fig. 2) probably decayed rather slowly. Consequently, cores taken from above this level should give later dates for marine incursion than those taken from below the 400 m depth contour.

It is important to note that the sea bed topography assumed for the calculations of retreat rates was reconstructed by allowing the present topography to achieve isostatic balance with an ice load that was derived from the UMO reconstruction of the 18,000 BP Laurentide ice sheet. This reconstruction allows the ice sheet to reach the edge of the continental shelf and it is clear from other papers presented at this conference that the Laurentide ice sheet during the late-Wisconsin may have extended no further than the northern end of the Laurentian Channel. This would imply thinner ice over the St. Lawrence lowlands and less isostatic depression. Moreover, the present topography is probably still to some extent depressed so that the basal profile assumed here is too deep. A shallower profile would give lower retreat rates so we may regard the present estimates as maximum probable values. Further improvement must await a better understanding of Laurentide ice margins, ice sheet reconstruction, crustal isostasy, late-Wisconsin climate and the dynamics of glacier sliding.

ACKNOWLEDGMENTS

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REFERENCES

BROTCHIE, J. F. and SILVESTER, R. (1969): On crustal flexure, *J. Geophys. Res.*, vol. 74, p. 5240-525.

CLARK, J. A. (1976): Greenland's rapid postglacial emergence. A result of ice-water gravitational attraction, *Geology*, vol. 4, 310-312.

GADD, N. (1975): Quaternary stratigraphy on the St. Lawrence lowlands, *Quaternary Stratigraphy Symposium*, York Univ., Toronto.

HUGHES, T. (1975): *West Antarctic ice streams*, ISCAP Bulletin No. 4, Inst. for Quaternary Studies, Univ. of Maine, Orono.

HUGHES, T., DENTON, G. H. and GROSSWALD, M. G. (in press): Was there a late-Würm Arctic ice sheet?, *Nature*.

KAMB, B. (1970): Sliding motion of glaciers: theory and observation, *Rev. Geophys. Space phys.*, vol. 8, No. 4, p. 673-728.

MÖRNER, N.-A. and DREIMANIS, A. (1973): The Erie interstade, *Geol. Soc. Amer.*, Mem. 136, p. 107-134.

SUDGEN, D. E. (1977): Reconstruction of the morphology, dynamics and thermal characteristics of the Laurentide ice sheet at its maximum, *Arct. and Alp. Res.*, vol. 9.

THOMAS, R. H. (1973): The creep of ice shelves: theory, *and* The creep of ice shelves: interpretation of observed behaviour, *J. Glaciol.*, vol. 12, No. 64, p. 45-53 and p. 55-70.

——— (1976): Thickening of the Ross ice shelf and equilibrium state of the West Antarctic ice sheet, *Nature*, vol. 259, No. 5540, p. 180-183.

WEERTMAN, J. (1957): On the sliding of glaciers, *J. Glaciol.*, vol. 3, No. 21, p. 33-38.

——— (1974): Stability of the junction of an ice sheet and an ice shelf, *J. Glaciol.*, vol. 13, No. 3, p. 3-11.

QUESTIONS AND COMMENTS

I. A. BROOKES:

"What evidence would you like to see provided by the field workers as input to your models?"

R. H. THOMAS:

"Firstly, the maximum late-Wisconsin limits of ice. Second, the isostatic depression in Laurentian channel or at least ice thickness there. Third, bottom sediment information from Laurentian Channel. Evidence of waterlaid tills overlain by silts would be nice. Finally, evidence of bottom fauna, or rather lack of fauna, since it is not likely that the sea floor would be very habitable beneath an ice shelf. But explorations beneath the Ross Ice Shelf will give us more on this."

A. DREIMANIS:

"What would be the minimum depth at the grounding line for the development of an ice shelf?"

R. H. THOMAS

"An ice shelf can, under appropriate circumstances, form at almost any depth. However, a calving bay will start to form if the sea depth exceeds a critical value that depends on the size of ice sheet, snow accumulation rate, ice temperature, basal sliding conditions, and presence or absence of a confined ice shelf. For a large ice sheet like the Laurentide this critical depth is about 400 m; for a smaller, but active ice sheet *without* a bounded ice shelf, the critical thickness varies approximately as the 4th root of the catchment area."