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Maria Rosa Massa-Esteve

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Aristarco de Samos. Acerca de los tamaños y las distancias del Sol y del la Luna. Estudio preliminar, revisión del texto griego y traducción al castellano by Christián Carman and Rodolfo P. Buzón

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Reviewed by
Maria Rosa Massa-Esteve*
Universitat Politècnica de Catalunya
m.rosa.massa@upc.edu

Aristarchus of Samos (*ca* 310–210 BC), who was born on the island of Samos, was a disciple of Strato of Lampsacus, the third director of the Lyceum, the school founded by Aristotle [Wall 1975; Bowen and Goldstein 1994; Stahl 2008]. Chronologically, Aristarchus is located between Euclid (*ca* 300 BC) and Archimedes (287–212 BC). Almost nothing is known of his life, and the little information available is determined by the citations found in later texts, such as Ptolemy's *Almagest* (AD 150) and Copernicus' *De revolutionibus orbium coelestium libri VI* (1543), as well as by the work that he himself left us, *On the Sizes and Distances of the Sun and the Moon*.

This work by Aristarchus belongs to ancient science, which may be considered in three fields of development:

- mathematics, especially that focused on the geometry in Euclid's *Elements* or Archimedes' works;
- astronomy, understood as a science that studies all the objects observed in the sky (i.e., the fixed stars, the Sun, the Moon, and the five planets) and their (apparent) movements; and, finally,
- ancient physics as represented by Aristotle's works.

Mathematics and astronomy are closely related in Aristarchus' work, and geometry is one of the essential tools that it uses for measuring the heavens,

* Maria Rosa Massa-Esteve holds a PhD in Science (Mathematics), History of Science program, from the Universitat Autònoma de Barcelona (1998). She is currently a full professor of Applied Mathematics in the Department of Mathematics at the Universitat Politècnica de Catalunya, where she is also an investigator of the Research Group on the History of Science and Technology (GRHCT). This research is included in the project PID2020–113702RB–I00 of the Spanish Ministerio de Ciencia e Innovación.

both for the geometric figures and their properties as found in Euclid's *Elements*.

In his work, Aristarchus starts from six hypotheses regarding the sizes and distances of the Sun and Moon and, by means of 18 propositions, he proves three theses:

(1) The distance from the Earth to the Sun is greater than 18 times but less than 20 times the distance from (the Earth) to the Moon.
(2) The diameter of the Sun has the same ratio to the diameter of the Moon (as obtains between their distances).
(3) The diameter of the Sun has a greater ratio to the diameter of the Moon than 19 has to 3, but less than 43 has to 6.

According to Pappus, Aristarchus' treatise was included in a collection of texts belonging to a course called the *Little Astronomy*, which also included *On the Sphere in Motion* by Autolycus of Pitane (*ca* 320 BC), the *Optics* and *Phenomena* by Euclid, and the *Sphaerics* and *De diebus et noctibus* by Theodosius (*ca* 107–43 BC), among other works [Ver Eecke 1982, 2.369]. The *Little Astronomy* was an introductory course to advanced astronomy that was in fact represented by Ptolemy's *Almagest*, originally known as the *Mathematical Composition*. Aristarchus' work has come down to us in copies that conserve each proposition along with their demonstrations and figures. The text thus enables us to gain insight into the geometry of Euclid's time. It is, therefore, important to have good translations of this work, such as that found here in the book by Carman and Buzón under review.

Acerca de los tamaños y las distancias del Sol y de la Luna (2020) is the second translation into Castilian of the work by Aristarchus and the first translation into that language from the original Greek text. It consists of 321 pages and is divided into two parts and two appendices, in one of which the authors describe the mathematical tools that are used in Aristarchus' work and in the other they deal with the list of manuscripts consulted. The book begins with a prologue by Beate Noack and a small introduction consisting of only two pages, and it closes with an exhaustive bibliography.

The first part of the book, titled "Study of the Work", consists of 90 pages and is divided into three sections. The first provides brief details about Aristarchus the man; the second presents a very interesting overview of the calculations of the distances and sizes of the Sun and the Moon made by several authors from before Aristarchus' time up to Halley; and the third section consists of 70 pages on the contents of Aristarchus' work and its structure, interpreting his propositions by means of modern mathematics.

The second part is the Castilian translation and consists of 134 pages, which include a comparison with other editions and translations. In fact, Aristarchus' text has had many translations. A translation from Greek to Arabic was made by Luqa al-Balabakki, who died in 912. Later, Nasir al-din al-Tusi (1201–1274) reviewed all the books in the *Little Astronomy*. The first printed edition of the work was a Latin translation by George Valla in 1488 (second edition: Venice, 1498). Various translations followed after this date, one into Latin by Commandino (1572). There was an edition of the Greek text by John Wallis (Oxford, 1688) and another Greek with Latin translation by Fortia d'Urban (Paris, 1810). This was followed by a French translation by the same author in 1823. In Oxford, in 1913, there was published a Greek edition with English translation by Thomas Heath, and in 2007, a translation into Castilian of Commandino's Latin version by Maria Rosa Massa-Esteve.

The enduring interest in Aristarchus' text during the medieval period, and the proliferation of articles about and translations of this text in modern times, may be seen as symptomatic of a continuous fascination with a text that in its time contributed to astronomy and to the measurement of the distances between the Earth, Sun, and Moon. In the present Castilian translation, the Carman and Buzón have used 30 manuscripts, all the editions of the original text in Greek, and all the translations into Latin and modern languages. All the texts are quoted in the margins of the translation, which also contains 261 footnotes. The translation follows the standard rules, making this ancient text easily accessible to a Castilian reader. Moreover, the constant comparisons with other editions provide expert investigators with access to the different translations of this source and are thus the strong point of the book.

A comprehensive assessment of this work on astronomy should take into account the close relationship between the beginnings of astronomy and the origins of trigonometry, an approach that contributes to a better understanding of the text. However, the modern mathematical interpretation of propositions that the authors make in the second part of the book does not assist in understanding the mathematics that Aristarchus actually used. In his work, Aristarchus poses problems of plane geometry by cutting the spheres of the Sun and the Moon into great circles. In order to solve the geometric problems, it was necessary to examine the relationships, now regarded as trigonometric, between the angles and the sides of a triangle. The angles are expressed as fractions of right angles, and Aristarchus wrote the ratios (now called trigonometric) as ratios between the sides of the triangles. The ratios should not be interpreted as quotients, as the authors of the book suggest: “Towards the quotient of the sizes of the Moon and the Sun...” [88]. The text is from 300 BC and this interpretation conceals Aristarchus’ achievements. These ratios allowed him to determine the upper and lower bounds of the value sought. The propositions that Aristarchus relies on are found mostly in Euclid’s *Elements*. Eudoxus’ theory of proportions from book 5 of the *Elements* is used throughout, and its properties of inverting, alternating, composing, and multiplying are applied for both equal and unequal proportions. Aristarchus implicitly bases himself on other relations (which for us are trigonometric) as if he already knew them or considered them trivial.

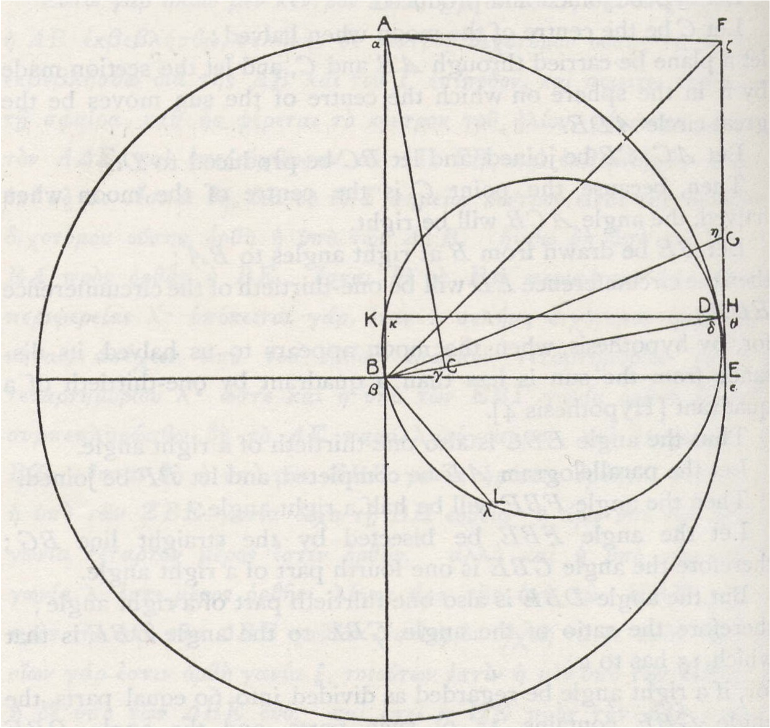

Figure 1. Aristarchus, *On the Sizes and Distances* prop. 7

An example can be seen in the demonstration of proposition 7, where Aristarchus states that the distance from the Earth to the Sun is greater than 18 times, but less than 20 times, the distance from the Earth to the Moon. Aristarchus constructs a right triangle, with vertices at the centers of the Earth (B), the Moon (C), and the Sun (A), with given angles or those known by observation [see Figure 1]. As the Moon appears to be divided into two semicircles at the quarters, $\angle BCA$ is 90°, $\angle ABC$ is 87° (by observation), and $\angle CAB$ is 3°. In fact, Aristarchus shows that

$$\frac{1}{18} > \sin 3^\circ = CB{:}AB > \frac{1}{20},$$

where CB is the distance of the Moon to the Earth and AB, the distance of the Sun to the Earth, and where the ratio of the distances is interpreted as the sine of the angle complementary to that between them.

With this approach, it is necessary to highlight the four mathematical strategies required for the development of the proof of the first inequality:

- the passage from the analysis of the problem of the triangle Sun-Earth-Moon to a similar triangle;
- the use of the relationship, as if it was trivial, between the tangents (current expression) and the angles ($\tan\alpha : \tan\beta > \alpha:\beta$, with the angles α, β of the first quadrant, and $\alpha > \beta$);
- the establishment of a proportion between the segments that determines the bisector of an angle and the sides of the triangle (applying proposition 6.3 of the *Elements*); and finally
- the approximation of $\sqrt{2}$ by 7:5.

At the end, Aristarchus transfers the result obtained in the triangle similar to the initial triangle ABC or the Sun-Earth-Moon triangle and concludes that $AB > 18CB$.

If we compare our interpretation with the mathematical interpretation made by the authors of this book ("It takes several steps, which we will not describe here" [85]), we find two important weaknesses: the bisector theorem from Euclid's *Elements* and Aristarchus' approximation of the irrational number by a ratio ($2 = 50{:}25 > 49{:}25$) and of $\sqrt{2}$ by 7:5. The same holds for the second inequality.

It should not be forgotten that Aristarchus' text is currently useful as an instrument for teaching mathematics. The use of the history of mathematics as an implicit and explicit resource makes it possible to improve the teaching of mathematics and the comprehensive training of students [Massa-Esteve 2003, 4–5]. History may serve as an explicit resource to introduce or understand better certain mathematical concepts through the analysis in the classroom of selected historical texts. Aristarchus' text provides highly instructive passages of geometry for use in the classroom [Massa-Esteve 2005, 95–101]. The text that has been tried and tested with high school and university students leads us to emphasize two fundamental ideas: the application of trigonometry to the calculation of distances and the relationship of trigonometry with its base tool, geometry.

Above and beyond mathematical ideas, the interest of Aristarchus' work also lies in the presentation of a rigorous method of calculating the relative Earth-Sun and Earth-Moon distances, which in 230 BC contributed to the extension of astronomical knowledge. This new Castilian translation provides a good source for understanding astronomy in the Hellenistic period.

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