Laval théologique et philosophique

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Volume 20, numéro 1, 1964

URI : https://id.erudit.org/iderudit/1020052ar
DOI : https://doi.org/10.7202/1020052ar

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Éditeur(s)

Laval théologique et philosophique, Université Laval

ISSN

0023-9054 (imprimé)
1703-8804 (numérique)

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Citer cet article

Felt, J. W. (1964). Mach's Principle Revisited. *Laval théologique et philosophique*, *20*(1), 35–49. https://doi.org/10.7202/1020052ar

Mach's Principle Revisited

Greek intellectual curiosity bore philosophy and science as twins, and it was centuries before anyone could tell them apart. It is no surprise, then, that philosophy should have fathered some of modern physics' most fundamental concepts. One of these has come to be known as *Mach's principle*. It began in germ with the epistemology of Bishop Berkeley, was developed by Mach, advocated by Einstein, and today commands considerable attention among physicists. It has never escaped the shadow of suspicion, however, since it has for eighty years eluded all attempts to define it with mathematical precision.

Because this rather enigmatic principle still bears strong features of its philosophical lineage, the philosopher finds legitimate interest in peering over the fence to see how it has fared since the days of Berkeley. He may also find philosophical relevance in its cosmic implications, as they have been gradually uncovered by physics.

I. Newtonian Absolute Space

Newton's idea of absolute space — an idea which pervades all of mechanics as Newton viewed it — is the beginning of the story. The first of Newton's familiar laws of motion, that "*every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it*,"[1] presupposes a definite coordinate system with reference to which the body's state of rest or motion is described. The spatial and temporal reference frames being presupposed are explained in detail in the first Scholium. Newton distinguishes absolute, true, and mathematical space from that which is relative, apparent, and common. "Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute space." (P.6.) An analogous distinction holds for time, place, and motion. Absolute or true motion thus consists in the motion of a body from one absolute place to another, though such motion is not directly apparent to the senses. For

1. *Sir Isaac Newton's Mathematical Principles of Natural Philosophy . . .*, trans. Andrew Motte, ed. Florian Cajori (Berkeley : University of California Press, 1946), p.13. Throughout this article italics in direct quotations are found in the originals unless otherwise noted.

For an eye-opening view of the implications packed into Newton's deceptively simple statement, see N. R. HANSON, "The Law of Inertia : A Philosopher's Touchstone," *Philos. of Science*, XXX (April, 1963), pp.107-121.

absolute space is by its very nature insensible ; yet the laws of motion as well as philosophical reflection convince us of its reality. " Because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them . . . But in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them."[2]

Because of the insensibility of absolute space, one cannot in the Newtonian system distinguish absolute from relative unaccelerated motion. For that matter, one cannot physically distinguish such motion from rest. In the case of rotational motion, however, Newton argues that absolute rotation is experimentally discernible by reason of the appearance of centrifugal forces. Thus he suggests : " If two

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2. *Principles*, p.8. Despite the pains which Newton took to clarify the sense in which he understood his absolute reference frames, there is today not complete agreement about his meaning. Stephen Toulmin, for instance, holds that Newton's absolute space and time are pure constructs postulated for the sake of the logical structure of his system. ("Criticism in the History of Science : Newton on Absolute Space, Time, and Motion, I," *Philos. Rev.*, LXVIII [January, 1959], pp.1-29.) In distinguishing absolute from relative space Newton is thought to have been employing, more or less consciously, a distinction that is a commonplace in modern axiomatic systems, "that between mathematical variables considered only for their place in a formal system and the same variables as figuring in applications of the system" (p.14). E. W. Strong has adopted a somewhat similar view. ("Newton's 'Mathematical Way'," *Jour. Hist. Ideas*, XII [1951], pp.90-110.) He distinguishes Newton's personal belief in the reality of absolute space from the purely postulational character he is thought to have given it in his system.

I cannot agree with these interpretations. In the first place, Newton appears to have surely believed in the real existence of absolute space and time. Absolute space for him was a physical reality, in the sense of being "out there," even though it was not by nature observable. It was more than just a mathematical construct.

But can one, then, suppose with Strong that although Newton personally believed in the physical reality of absolute space and time, yet for want of direct empirical confirmation he did not introduce them into his system except as non-empirical or "metaphysical" postulates ? There seems to be no solid evidence for this. Indeed, it is hard to believe that Newton thought that the basic insensibility of absolute space put it into the category of a metaphysical construct ; it is then equally hard to believe that he had Strong's neat distinction in mind when he wrote the *Principia*. He seems, rather, to have believed that his laws of motion were explicit expressions of laws actually operative in nature, and that absolute space and time, though not directly observable, were extramental principles on which the physical universe really depended. Inertia, it will be recalled, was for Newton a *vis*, a kind of force really operative within a body, by which it resists attempts to change its state. But its state is a function of a reference frame, and it would be strange if Newton regarded this physically operative force as depending in its operation on a reference frame which was not as physically real as the force itself.

There is no scandal if even Newton, living when he did, did not altogether succeed in breaking away from the custom of mixing some philosophy into his physics ; of trying to attain something of what nature is, as well as of how nature operates.

For corroboration of this view, see Max Jammer, *Concepts of Space* (Cambridge : Harvard University Press, 1957), p.100, and H. G. Alexander (ed.), *The Leibniz-Clarke Correspondence* (Manchester : Manchester University Press, 1956), Introduction, pp. xxxix-xl.

globes, kept at a given distance one from the other by means of a cord that connects them, were revolved about their common centre of gravity, we might, from the tension of the cord, discover the endeavor of the globes to recede from the axis of their motion, and from thence we might compute the quantity of their circular motion." (P.12.)

His more memorable example, however, is that of the water-bucket experiment (pp.10-11). If a vessel containing water is suddenly set into rotation, the water at first remains stationary and with a plane surface. As the water gradually takes up the rotation, however, its surface becomes increasingly concave, reaching a maximum when the water revolves at the same rate as the vessel. If the motion of the vessel is then suddenly arrested, the water at first continues to rotate within it and to retain its concavity. Both at this time and at the beginning of the experiment there is an equivalent amount of relative motion between the bucket and the water ; yet at the beginning the water's surface is plane, at the end it is concave. Hence, Newton argues, the appearance of centrifugal forces acting on the particles of water (as indicated by the concavity) cannot be due to relative motion but rather to rotation of the water with respect to absolute space.

Whether or not this conclusion follows from the premises — and Mach was to argue strenuously that it does not — many have felt that there is someting of the *deus ex machina* about Newton's absolute space. Could not motion (as Berkeley and Mach contended) be described just as effectively without positing any such absolute? Furthermore, since absolute space is in principle insensible (due to its perfect homogeneity), the only way its existence could be physically verified is by appealing to those very effects which it is postulated in order to explain. And it makes a strange explanation : an infinite, homogeneous, massless, and undetectable medium is supposed to condition the motions of finite, massive bodies, constraining them to move according to fixed laws, yet without in turn being conditioned by them. Taken purely as description, such an account seems to violate the ideal of economy of postulates. Taken as explanation, it merely sweeps the problem under the rug : bodies act the way they do because of the presence of an undetectable principle postulated precisely to explain why bodies act the way they do.

Actual measurement of the rotation of the earth provides a clear example of this method at work. Measurement by means of a Foucault pendulum and measurement by observation of the fixed stars (galaxies) give identical results. The simplest explanation for this agreement would be to suppose that the inertia of the pendulum is somehow dependent on the galaxies, hence the identical results are to be expected. Newtonian mechanics, however, gratuitously introduces a third and unobservable element with respect to which the states of motion of the pendulum and the galaxies are separately explained. If the former description is workable, however, the value of the latter becomes questionable.

There are still other drawbacks of the Newtonian system which concern us. From Newton's second law of motion, as well as from his definition of inertia, it is clear that mass and inertia are complementary terms. A body's resistance to a change of its state of motion is exactly proportional to its mass. But a similar mutual relationship exists between mass and the force of gravity : in his definition of mass Newton states that mass " is known by the weight of each body, for it is proportional to the weight " (*Principles*, p.1). These distinct characterizations of mass logically define formally distinct quantities, inertial mass and gravitational mass. Their numerical equivalence in actual fact has been established by experiments accurate to one part in ten billion, yet this remarkable equivalence of formally distinct quantities remains totally without explanation in the Newtonian system.[1]

Again, Newton takes great pains to base his system on immediate evidence, avoiding appeal to the " occult qualities " which abounded in the Aristotelian school.[2] Yet at the very core of his system he seems to have had a curious relapse in this regard. This is his treatment of inertia itself. It is a kind of force or *vis* by which a body tends to resist a change of its state of rest or motion with respect to absolute space. Nowhere does Newton reveal any discomfort at appealing to so mysterious an activity to explain so much ; yet this is surely an even more puzzling phenomenon than gravity. It is easier to conceive some unseen force by which two material bodies act on each other than to conceive a force by which a material body resists a change of its state relative to an immaterial, insensible, infinite reference frame. Newton had said of the Aristotelians : " To tell us that every Species of Things is endow'd with an occult specifick Quality by which it acts and produces manifest Effects, is to tell us nothing." (*Opticks*, p.401). But to tell us that a body is hard to set in motion because of a force relation it has to an undetectable space, is to come dangerously close to the same sort of vacuous explanation.

All these deficiencies of the Newtonian system, however, are remedied immediately if Mach's principle is accepted.

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1. Concerning R. H. Dicke's recent confirmation of the original experiment by Eötvös, see his "Eötvös Experiment," *Scientific American*, December, 1961, pp.84-94. The very simplest form of such an experiment consists in the observation that bodies of equal volume but different masses nevertheless fall at the same rate. This would not be true if gravitational and inertial mass were not either equivalent or strictly proportional. Einstein was most insistent on the need for establishing a formal identity between the two masses. See his *Meaning of Relativity*, 5th ed. (Princeton : Princeton University Press, 1955), pp.56-57.

2. Sir Isaac NEWTON, *Opticks* (New York : Dover Publications, 1952), Bk III, p.401.

II. *Criticism of Newton's Absolutes*

When in 1710 George Berkeley published his *Treatise concerning the Principle of Human Knowledge*, he was inevitably led to comment on Newton's ideas concerning space and time. He deals with them also in his later essay, *De Motu*.

Berkeley, naturally enough, held that the motion of a body should be regarded purely in relation to other bodies, not to absolute space. "It doth not appear to me that there can be any motion other than *relative* ; so that to conceive motion, there must be at least conceived two bodies, whereof the distance or position in regard to each other is varied. Hence if there was one only body in being it could not possibly be moved. This seems evident, in that the idea I have of motion doth necessarily include relation."[1]

In the very next paragraph, though, he softens his relativism by associating true motion with the force causing the motion. "But though in every motion it be necessary to conceive more bodies than one, yet it may be that one only is moved, namely that on which the force causing the change of distance is impressed . . . I ask anyone, whether in his sense of motion as he walks along the streets, the stones he passes over may be said to *move*, because they change distance with his feet ?" Berkeley uses this distinction to explain Newton's bucket experiment without appealing to absolute space. At the beginning of the experiment the water is not in a true state of (relative) motion, since the frictional force from the walls of the container has not yet been effectively applied (*ibid.*, p.92).

The remark, however, in which Berkeley touches most nearly on the principle which Mach was to enunciate a century and a half later concerns the substitution of "the sky of the fixed stars" for the absolute space of Newton. He argues that, since motion is purely relative, one cannot conceive any motion at all of a single sphere, if one supposes all other bodies have been annihilated.

Then let two globes be conceived to exist and nothing corporeal besides them. Let forces then be conceived to be applied in some way ; whatever we may understand by the application of forces, a circular motion of the two globes round a common centre cannot be conceived by the imagination. *Then let us suppose that the sky of the fixed stars is created ; suddenly from the conception of the approach of the globes to different parts of that sky the motion will be conceived.* That is to say that since motion is relative in its own nature, it could not be conceived before the correlated

1. "Principles of Human Knowledge," *The Works of George Berkeley*, eds. A. A. Luce and T. E. Jessop (London : Thomas Nelson and Sons, 1948-57), II, p.91, par.112. All references to Berkeley's works will be to this edition.

bodies were given. Similarly no other relation can be conceived without correlates.[1]

A century and a half later Ernst Mach, " Professor of the History and Theory of Inductive Science " in the University of Vienna, renewed this criticism with interest. In his *History and Root of the Principle of the Conservation of Energy* (1872) and his *Science of Mechanics : A Critical and Historical Account of Its Development* (1883 and later) he subjected the whole Newtonian system to a searching criticism. We shall confine our attention to those parts which bear on inertia and coordinate systems.

In Mach's view, the most primitive fact of experience on which mechanics is to be based is not the law of inertia (Newton's First Law of Motion) but rather the force-acceleration ratio expressed in the Second Law. This already contains within itself the law of inertia (First Law) since, if acceleration is proportional to the force impressed, it is already clear that no acceleration will occur if the body is left to itself. It is superfluous to add the law of inertia, as if inertia and force (or the force-acceleration law) were distinct facts of experience.

As for Newton's absolutes, Mach remarked : " It would appear as though Newton . . . stil stood under the influence of the mediaeval philosophy, as though he had grown unfaithful to his resolve to investigate only actual facts."[2] For Mach, absolute space and time were pure constructs, fictions of the mind, not part of our experience ; no one is justified in pretending otherwise.[3] Real motion is essentially relative to bodies, so that it is not possible to talk about, let alone deal with, absolute motion. We have our concepts only through sense experience, and our sense experience is always of other bodies in relation to which the motion takes place, never of insensible absolute space. It is meaningless to talk of the motion of a body as if it were independent of the existence of other bodies.

1. " *De Motu*," IV, p.47, par.59 ; italics added.

2. *The Science of Mechanics*, trans. Thomas J. McCormack, 3rd Engl. ed. (Chicago : Open Court Publishing Co., 1907), p.223 [272]. The Appendix to this edition, containing several valuable remarks relevant to Mach's principle, does not appear in the latest (6th) English edition (La Salle, Illinois : Open Court, 1960). Page references will therefore be given primarily to the older edition, with corresponding pages of the 1960 edition given, where possible, in square brackets.

3. *Ibid.*, p. 229 [280]. When discussing the same point in the Appendix, Mach adds : " I flatter myself on being able to resist the temptation to infuse lightness into a serious discussion by showing its ridiculous side, but in reflecting on these problems I was involuntarily forced to think of the question which a very estimable but eccentric man once debated with me as to whether a yard of cloth in one's dreams is as long as a real yard of cloth, — Is the dream yard to be really introduced into mechanics as a standard of measurement ? " (P.569*n.*)

As for rotational motion, centrifugal forces do not prove the existence of an absolute space. For if it be urged that such forces would obviously not arise if the water in the bucket stood still and everything else, including the heavenly bodies, rotated around it, the objection clearly assumes just what is to be proved, namely, the existence of absolute space with respect to which the water can be said to be at rest while the stars do the moving. Otherwise how is one to distinguish the two cases, or even to speak of two different cases? In any event, how could one know the outcome without actually trying it? "Can we fix Newton's bucket of water, rotate the fixed stars, and *then* prove the absence of centrifugal forces?

"The experiment is impossible, the idea is meaningless, for the two cases are not, in sense-perception, distinguishable from each other. I accordingly regard these two cases as the *same* case and Newton's distinction as an illusion." (*Ibid.*, p.543.)

Even the negative result of the relative motion between the bucket and the water at the beginning of Newton's experiment is not a conclusive argument against the function of purely relative motions in producing centrifugal forces.

Newton's experiment with the rotating vessel of water simply informs us, that the relative rotation of the water with respect to the sides of the vessel produces *no* noticeable centrifugal forces, but that such forces *are* produced by its relative rotation with respect to the mass of the earth and the other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us.[1]

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1. *Ibid.*, p.232 [284]. Of interest in this connection is the following paragraph from the Appendix :

" The most captivating reasons for the assumption of absolute motion were given thirty years ago by C. NEUMANN. . . . If a heavenly body be conceived rotating about its axis and consequently subject to centrifugal forces and therefore oblate, nothing, so far as we can judge, can possibly be altered in its condition by the removal of all the remaining heavenly bodies. The body in question will continue to rotate and will continue to remain oblate. But if the motion be relative only, then the case of rotation will not be distinguishable form that of rest. All the parts of the heavenly body are at rest with respect to one another, and the oblateness would necessarily also disappear with the disappearance of the universe. I have two objections to make here. Nothing appears to me to be gained by making a meaningless assumption [absolute motion] for the purpose of eliminating a contradiction. Secondly, the celebrated mathematician appears to me to have made here too free a use of intellectual experiment, the fruitfulness and value of which cannot be denied. When experimenting in thought, it is permissible to modify *unimportant* circumstances in order to bring out new features in a given case ; but it is not to be antecedently assumed that the universe is without influence on the phenomenon here in question. If it is eliminated and contradictions still result, certainly this speaks in favor of the importance of relative motion, which, if it involves difficulties, is at least free from contradictions." (P. 572).

Having established on epistemological grounds that motion is not understood in terms of an absolute space but rather in terms of other bodies, Mach makes it explicit that it is the entire universe of bodies with which one has to deal : " When . . . we say, that a body preserves unchanged its direction and velocity *in space*, our assertion is nothing more or less than an abbreviated reference to *the entire universe*." (*Ibid.*, p.233 [285-286].) *This last statement is essentially just what has come to be known as Mach's principle.*

This principle is physical, not merely epistemological (as was Berkeley's reference to the sky of the fixed stars) ; Mach immediately begins seeking an expression for it in terms of masses and accelerations (hence also of inertia). This dynamic aspect stands out with particular clarity in a passage from Mach's earlier work :

> Obviously it does not matter whether we think of the earth as turning round on its axis, or at rest while the celestial bodies revolve round it. Geometrically these are exactly the same case of a relative rotation of the earth and of the celestial bodies with respect to one another . . .
>
> But if we think of the earth at rest and the other celestial bodies revolving round it, there is no flattening of the earth, no Foucault's experiment, and so on — at least according to our usual conception of the law of inertia. Now, one can solve the difficulty in two ways : Either all motion is absolute, or our law of inertia is wrongly expressed. Neumann preferred the first supposition, I, the second. The law of inertia must be so conceived that exactly the same thing results from the second supposition as from the first. By this it will be evident that, in its expression, regard must be paid to the masses of the universe.[1]

But how are we to give mathematical expression to this new viewpoint ? Mach suggests that the usual statement that the direction and velocity of a mass μ in space remain constant in the absence of external forces, be replaced by the statement that the mean acceleration of μ with respect to the other bodies $m, m', m'', \ldots,$ of the universe, at distances $r, r', r'', \ldots,$ is zero. That is, that $d^2(\Sigma mr/\Sigma m)/dt^2=0$. If we have taken a sufficient number of distant bodies into account, then the mutual influence of neighboring bodies will make a negligible contribution to the law of their behavior, which is in accord with our experience. An obvious difficulty, however, is that we cannot actually complete the summation, both because of the very imperfect state of our astronomical knowledge, and because of the enormousness of the undertaking. Hence it is necessary in calculations to resort to some sort of smoothed out model which will give a workable approximation.

It is unnecessary, Mach concludes, to refer the law of inertia to absolute space, since we can much more meaningfully refer it

1. *History and Root of the Principle of the Conservation of Energy*, trans. Philip E. B. Jourdain (Chicago : Open Court Publishing Co., 1911), pp.76-77.

to the sum of the actual bodies in the universe. This reference supposes the interdependence of all the masses in the universe, since the law of acceleration, which is the basic fact of our experience and which we extend to all masses, itself involves the law of inertia (*Mechanics*, p.236 [288]).

It may be asked how much of his principle Mach owed to Berkeley. Both Sciama and Whitrow regard Mach's work as only a slight elaboration of Berkeley's initial suggestion concerning the fixed stars as a fundamental reference system.[1] Yet there is a greater difference in the viewpoints of the two men than verbally appears. Berkeley's approach is almost purely epistemological ; from the viewpoint of mechanics his description is entirely kinematic. He is interested in how we recognize motion, not in how motion comes about. He introduces the fixed stars simply in order to render the motion in question conceivable. Mach's outlook, on the other hand, is more that of a physicist. He employs dynamic as well as purely kinematic description. In the enunciation of his principle he is explicitly concerned with the relation of forces to masses ; in one place he even speaks of the heavenly bodies as belonging to " the causal nexus of the phenomenon " of inertia (*Conservation of Energy*, p.79). In the light of this dynamic aspect, Berkeley's remark is seen to bear only an imperfect resemblance to Mach's principle.[2]

III. *Mach's Principle in Modern Theories*

Several attempts have been made to incorporate Mach's principle into the mathematical structure of general physical theories. It is an attractive prospect. If Mach was right in supposing that the masses in the universe are responsible for inertial effects, then, since all masses are assumed to act on each other by gravitational attraction, and since gravitational mass and inertial mass are numerically equivalent, the obvious inference arises that inertia is really a gravitational effect. In that case, inertial mass and gravitational mass become formally as well as numerically identical. Furthermore, the total amount of matter in the universe would determine the inertial or gravitational constant, so that the value of this constant would furnish information about the total mass of the universe.

The best known attempt to embody Mach's principle in a general theory is Einstein's general theory of relativity. In his student

1. D. W. Sciama, *The Unity of the Universe* (Garden City, New York : Doubleday & Company, 1959), p.115 ; G. J. Whitrow, *The Structure and Evolution of the Universe* (New York : Harper and Brothers, 1959), p.62, and in his " Berkeley's Philosophy of Motion," *Brit. Jour. for Philos. of Science*, IV (1953), pp.37-45. See also K. R. Popper, "A Note on Berkeley as Precursor of Mach," *ibid.*, pp.26-36.

2. Jammer makes the same point in his *Concepts of Space*, p.107.

years Einstein had been profoundly influenced by Mach's *Science of Mechanics*. Though he later came to disagree with Mach's epistemological position, he was always inclined to accept Mach's principle. He appears to have first introduced it into general relativity not as a physical but rather as an epistemological and heuristic principle. In *The Foundation of the General Theory of Relativity* (1916) he introduced it as Mach's criticism of an epistemological flaw (that of absolute space, or, as Einstein preferred to put it, of preferred coordinate systems) in Newtonian mechanics. Einstein maintained with Mach that the behavior of bodies must be attributed not to a factitious cause (absolute space) but rather to the causal influence of the distant masses of the universe. Yet instead of immediately employing this as a principle of physical significance, Einstein refers to this whole argumentation as a " weighty argument from the theory of knowledge," and proceeds instead to infer the *principle of covariance : "The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion."* [1]

In *Cosmological Considerations on the General Theory of Relativity*, published the following year, Einstein treats Mach's principle in a more physical way. He there clearly spells out one of its necessary physical consequences : " In a consistent theory of relativity there can be no inertia *relatively to 'space,'* but only an inertia of masses *relatively to one another*. If, therefore, I have a mass at a sufficient distance from all other masses in the universe, its inertia must fall to zero." [2] When in 1932 Einstein abandoned his efforts to secure this result in general relativity, he also abandoned, though reluctantly, the attempt to incorporate Mach's principle fully into the general theory of relativity. In present formulations of general relativity, a body in an otherwise empty universe would nevertheless possess inertia.[3]

Recently there have been several fresh approaches to the problem of giving mathematical expression to Mach's principle. Of these,

1. " The Foundation of the General Theory of Relativity," *The Principle of Relativity*, trans. W. Perret and G. B. Jeffrey (New York : Dover Publications, no date given), p.113.

2. *Princ. Rel.*, p.180. It appears to have been this consideration which led Einstein to adopt a non-Euclidean geometry for his static model of the universe. He demonstrated that the only available boundary (infinity) conditions for a static universe whose space is quasi-Euclidean fail to satisfy this requirement of the complete relativity of inertia. (P. 183 ; see also his *Meaning of Relativity*, pp.98-99.) In order, then, to safeguard Mach's Principle he introduced into his gravitational equations the later-abondoned " cosmological term " which, if the cosmological constant λ has a positive value, has the effect of making the geometry of the universe closed, thus abolishing infinity at which the boundary difficulties occur.

3. See H. BONDI, *Cosmology*, 2nd ed. (Cambridge : Cambridge University Press, 1960), p.99. Mach's principle may nevertheless be regarded as underlying general relativity's principle of equivalence (the local equivalence of descriptions in gravitational or in inertial terms), since it implies that inertia is itself simply a gravitational effect.

the tentative theory suggested by the English cosmologist D. W. Sciama brings out with perhaps the greatest clarity the cosmic implications of the principle.[1] In this theory he elaborates a simplified version of a proposed more general theory which, by direct use of Mach's principle, would account for the inertial properties of matter.

Sciama's aim, therefore, is to construct a formalism which yields the inertial properties of matter as a mathematical consequence, rather than assumes them. To be satisfactory such a theory must also explain why the physical properties of the universe as a whole appear to be irrelevant to the laws of mechanics, a fact that has been one of the principal obstacles to the acceptance of Mach's principle. A physicist in a windowless laboratory could, in principle, develop all the essentials of Newtonian mechanics without ever knowing what is outside. What is more, it might seem as if it would make no difference to mechanics if the stellar universe were suddenly to pass out of existence. Many physicists still incline to agree with Eddington's remark : " We do not believe that if the heavenly bodies were all annihilated, it would upset the gyrocompass."[2] In particular, the apparently absolute and independent character of the inertial properties of bodies (an independence which seems to openly contradict Mach's principle) must be explained. Newton explained it by reference to absolute space, though he was unable to explain either the equivalence of inertial and gravitational mass, or the magnitude of the gravitational constant.

Sciama is led, by his simplifying assumptions, to describe the gravitational field by means of Maxwell's electromagnetic equations, interpreting them, however, as purely gravitational instead of electromagnetic. The results are impressive as far as they go :

(1) The theory yields an approximate mathematical relation between the actual strength of gravity (as expressed by the gravitational constant) and the total mass of the universe. Such an equation could, in principle, be tested by observation.

(2) Inertial effects are predicted which are entirely gravitational in origin, so that the inertia of a test body would indeed tend to zero as all other bodies are removed.

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1. D. W. SCIAMA, " On the Origin of Inertia," *Monthly Notices of the Royal Astronomical Society*, CXIII (1953), pp.34-42. For other noteworthy theories see, for example, R. H. DICKE, " Gravitation without a Principle of Equivalence," *Rev. Mod. Phys.*, XXIX (1957), pp.363-376 ; Parry MOON and Domina Eberle SPENCER, " Mach's Principle," *Philos. of Science*, XXVI (1959), pp.125-134 ; C. BRANS and R. H. DICKE, " Mach's Principle and a Relativistic Theory of Gravitation, *Phys. Rev.*, CXXIV (1961), pp.925-935. This last article will be discussed below. R. H. Dicke has suggested still another approach, based on the hypothesis of certain " Machian fields," in " Cosmology, Mach's Principle and Relativity," *Amer. Jour. Phys.*, XXXI (July, 1963), pp.500-509.

2. Sir Arthur EDDINGTON, *Space, Time and Gravitation*, (New Yord : Harper and Brothers, 1959), p.153.

(3) The contribution of matter to local inertia is seen to fall off only as the inverse first power of the distance. But since the amount of matter at a given distance in the universe (assuming uniform distribution) is proportional to the *square* of that distance, it follows that the main contribution to local inertia comes from far distant matter. In fact, ninety-nine percent of local inertia arises from matter farther away than 100-million light-years. The Earth's fractional contribution to inertia is only one 1,000-millionth ; that of the Sun is one 100-millionth ; that of the Milky Way is one 10-millionth. This explains the apparent irrelevance of the universe as a whole to the laws of mechanics. Paradoxically, the entire universe is so relevant that it seems irrelevant : the contribution to local inertia of any bodies within the scope of our experiments is so slight as to be undetectable.[1]

(4) The new theory perfectly satisfies the principle of equivalence. Exactly the same effect is produced for the observer in Einstein's hypothetical elevator whether a gravitating mass is suddenly placed near the elevator or the elevator is suddenly accelerated with respect to the rest of the universe.

(5) It is a mathematical consequence of the new theory that the gravitational force must be attractive, whereas the sigh of the field is undetermined in general relativity.

(6) In the case of uniform relative rotation between a test body and the rest of the universe, the forces analogous to the electric and magnetic forces predicted by Maxwell's theory turn out to be just the centrifugal and Coriolis forces which must be introduced into the Newtonian equations of motion when dealing with non-inertial systems. In the new theory, however, these forces are in no sense fictitious, but are real forces, gravitational in origin. Thus, in this theory, the water in Newton's bucket became concave precisely because it was in a state of rotation relative to the sum of the masses in the universe. More conveniently for purposes of calculation, the masses of the universe can be considered to rotate about the water, and to produce the concavity by gravitational induction.

In this same article Sciama expresses the intention of publishing later a more general version of his theory, freed from the many simplifying assumptions of the first. It would be expected to yield a precise mathematical relation between the gravitational constant and the mean density of matter in the universe. With the advance

1. Compare this position with that taken by P. W. BRIDGMAN, " Significance of the Mach Principle," *Amer. Jour. Phys.*, XXIX (January 1961), pp.32-36. In this article he develops the idea that " the universe of stars appears to have some connection with local phenomena for the paradoxical reason that it has no connection." The foregoing considerations on the dynamic aspect of Mach's principle imply that Mach would scarcely have indorsed Bridgman's interpretation.

of astronomy this equation should be susceptible of observational verification.

This hope, however, now appears unlikely of fulfilment. The equations to which Sciama has been led in the generalized theory turn out to be just Einstein's own gravitational equations, and the non-linearity of these equations denies us the derivation, at least by presently known methods, of an experimentally testable relation. What it comes to physically is this. The non-linearity is connected with the fact that the gravitational field radiated by a body represents a form of energy. Energy is equivalent to inertial mass according to Einstein's formula, $E=mc^2$. But inertial mass, by the nature of the theory, is identical with gravitational mass ; hence the gravitational field itself induces a secondary gravitational field ; and so on, indefinitely. In addition, the further the physical analysis is carried the more difficult it becomes to distinguish clearly between the radiating body and the radiated gravitational field. Thus the requirement of the complete relativity of inertia (e.g., that the inertia of a body fall to zero in the absence of all other bodies) loses its clear meaning.[1]

Another approach has recently been suggested by Carl Brans and R. H. Dicke.[2] Their theory consists in a generalization of general relativity which is thought to bring it into full accord with Mach's principle. The space geometry of such a system should depend uniquely upon the matter distribution in the universe. In order to achieve this result, a term was added to the ordinary variational principle of general relativity, selected in such a way as to make inertia a gravitational effect. It was also necessary to modify the form of general relativity's principle of equivalence, changing it from the "strong" form employed by Einstein (that the laws of physics as observed locally in a freely-falling laboratory are independent of the location of the laboratory in the universe), to the "weak" form (that there is a local equivalence of gravitational forces and accelerations) which alone has been verified by the experiments of Eötvös and of Dicke.[3]

The resulting field and boundary-value equations have not yet been worked out in full generality, but the authors seem justified in entertaining a cautious optimism. As for experimental verification, no conclusive results have as yet been obtained, largely due to our presently poor knowledge of stellar evolutionary rates. Since these are a sensitive function of the gravitational constant, their observation

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1. Sciama gave the substance of these results of his general theory in a lecture at Washington University, Saint Louis, in May, 1961. They are mentioned here with his permission.

2. C. Brans and R. H. Dicke, "Mach's Principle and a Relativistic Theory of Gravitation," *Phys. Rev.*, CXXIV (1961), pp.925-935.

3. See p. 38, note 1.

makes a test of the theory possible in principle. In another article, in which he discusses the progress made so far in making a cosmological test of the theory, Dicke states :

> We conclude that the data discussed are not yet sufficient to support strongly the Brans-Dicke theory. However, the theory provides the only physical explanation for the age discrepancies of galaxies, etc., that has so far appeared ... It seems certain that ... compelling evidence for or against the Brans-Dicke theory is not many years off.[1]

IV. *Retrospect*

The objections which have long stood in the way of a universal acceptance of Mach's principle are serious. One hesitates to quarrel with success, and the Newtonian system, based on absolute rather than relative motion, has been wonderfully successful. Furthermore, although the gravitational attraction between two masses can be demonstrated in the laboratory, no local experiment has yet shown any relation between local masses and inertia ; inertia appears to be independent of other masses. From the theoretical side, the lack of a clear-cut and verified mathematical expression for the principle, even after eighty years, discourages its ready acceptance. Finally, the incompatibility of the principle (in its full form) with general relativity can be regarded as a strong argument against the principle.

Replies are not lacking, however, to these objections. What a system explains may not be so significant as what it fails to explain. It is precisely the failure of the Newtonian system to explain inertia in general, and the factual equivalence of gravitational and inertial mass in particular, that constitutes a powerful argument for any new approach which would account for them in a natural way. As for the negative result of attempting to alter inertial effects in the laboratory, it cannot be effectively urged against Mach's principle, since it has been shown to be just what one would expect : if inertial effects are almost entirely due to masses in the universe lying at an immense distance, the effect of local masses will be experimentally undetectable. The difficulty in obtaining a mathematical expression for the principle is not surprising, since it necessarily involves the interlocking effects of all the bodies in the universe. Observational confirmation of any such expression has not yet been achieved, but it may be only a few years away. Finally, the incompatibility of the principle with general relativity may be regarded as arguing rather against present forms of relativity than against the principle itself. The Brans-Dicke theory is precisely a modification of the form of general relativity. In using the weak rather than the strong form of the principle of

1. R. H. Dicke, " Implications for Cosmology of Stellar and Galactic Evolution Rates," *Revs. Mod. Phys.*, XXXIV (1962), pp.110-122.

equivalence, it seems to be on a sounder footing of experimental evidence than does Einstein's version.

The elimination of undetectable absolute space, the formal identification of gravitational and inertial mass, and the explanation of inertia as a gravitational effect all argue strongly in favor of Mach's principle. Its cosmic implications as they have now become apparent, seem to deepen this argument : physicists, even in their laboratories, can no longer afford to ignore the rest of the universe.[1]

Neither, of course, can philosophers, and Mach's principle underscores this as well. As a purely physical principle, it enjoys only whatever status physicists may be pleased to give it ; they are obviously at liberty to refer motion to any coordinate system they please. Berkeley, however, was not free when he embraced its epistemological analogue. His metaphysics required it. And, in fact, the relating of apprehended motion to other bodies (ultimately to all other bodies) seems to be the only natural alternative to Newton's reference to a real but insensible absolute.

Of still greater importance for the philosopher is the almost incredible degree of dynamic solidarity revealed in the universe by Mach's principle. If Sciama's theory is anywhere near the truth, then the inertia exhibited by the pencil on one's desk is almost totally due to masses in the universe which lie beyond the reach of the two-hundred inch telescope ! A philosopher's whole attitude toward the universe, and his handling of interrelationships, are bound to be influenced by such a realization. This dynamic unity of the universe seems almost to repeat in physical terms Whitehead's insistent theme, " The whole world conspires to produce a new creation."[2]

James W. Felt, s.j.

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1. " Physicists should remember that the inertial-gravitational (or metric) field, and by implication cosmology, is basic to mechanics and concerns all branches of physics." (Dicke, " Implications," p.110.)

2. Alfred North Whitehead, *Religion in the Making* (Cleveland : The World Publishing Company, 1961), p.109. This is not to count Whitehead among the proponents of Mach's principle ; on the contrary, he explicitly rejected it in his early writings. In his *Enquiry concerning the Principles of Natural Knowledge* (1919), he says : " It has been asserted that after all the fixed stars are essential, and that it is the rotation relatively to them which produce the bulge [of the earth]. But surely this ascription of the centrifugal force on the earth's surface to the influence of Sirius is the last refuge of a theory in distress. The point is that the physical properties, size, and distance of Sirius do not seem to matter." (Cambridge : at the University Press, 1955, p.36.) Again, in *The Concept of Nature* (1920) he writes : " I cannot persuade myself to believe that a little star in its twinkling turned round Foucault's pendulum in the Paris Exhibition of 1861. Of course anything is believable when a definite physical connexion has been demonstrated . . . Here all demonstration is lacking in the form of any coherent theory." (Ann Arbor Books, University of Michigan Press, 1959, p.138.) The coherence of modern theories based on Mach's principle, and their explanation of why Sirius or any other twinkling star does not seem to matter, incline one to think that Whitehead would today revise his estimate of Mach's principle.