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### Article abstract

Network properties of national domestic airline Systems are examined and linked to causal factors such as levels of economic development, population size and distribution, topographic relief, and size of country. Graph theoretic indices are utilized to measure network characteristics and become the independant variables in regression analyses. The theoretical pitfalls of this method are highlighted by utilizing a path analytic framework to identify the degree of interrelationship among the dependent variables. Still, the graph theoretic method is deemed useful as a means of topologic analysis of network structures.

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# NATIONAL AIRLINE NETWORKS: A GRAPH THEORETIC ANALYSIS

by

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Most nations have either domestic or international commercial airline service. In many cases even the small developing nations have complex domestic commercial airway systems. Conversely, some technologically developed nations have rather limited networks. The complexities of these systems are not based solely on economic variables, but are the product of a series of causal factors. Indeed, the complexity of airline networks depends on determinants ranging from topographic relief to per capita income. It is the purpose of this paper to identify the fundamental national characteristics accounting for the nature of existing airline networks.

Using regression analysis, relationships between domestic airline networks and national characteristics will be formulated. This technique of network analysis has been used by researchers (Garrison and Marble, 1961; Kansky, 1963; Vetter, 1970) to ascertain the causal linkages between national economic and geographic characteristics and surface transport networks. The method is not, however, beyond reproach. Modifications will be suggested and in general, the technique will be evaluated.

The domestic airway systems of twenty-five countries <sup>1</sup> were reconstructed from listings in the Official Airline Guide. All connections with foreign cities were excluded even if there existed a more convenient connection between domestic cities via foreign airports. Several graph theoretic indices were derived from the resulting networks.

# GRAPH THEORY AND AIRLINE NETWORKS

A study of relationships between the structure of transportation networks and the regional characteristics requires measurement of the transport system. There is a necessity to identify, for instance, such network characteristics as the number of points served, directness of connections and overall complexity. Graph theory is selected as the method of translating

<sup>&</sup>lt;sup>1</sup> The twenty-five countries, corresponding to those used by Garrison and Marble, include Algeria, Angola, Bolivia, Bulgaria, Ceylon, Chile, Cuba, Czechoslovakia, Finland, France, Ghana, Hungary, Iran, Iraq, Malaysia, Mexico, Nigeria, Poland, Romania, Sudan, Sweden, Thailand, Tunisia, Turkey, and Yugoslavia

network characteristics to variables suitable for quantitative analysis. Moreover, the fundamental precepts of graph theory facilitate better understanding of network properties.<sup>2</sup>

Graph theory as a descriptive technique recognizes two fundamental types of networks: planar graphs and non-planar graphs. A planar graph is one in which the intersection of two edges (routes) is a vertex. Planar graphs are used for highway and railroad networks where crossings are at grade or are interconnected by interchanges. If two intersecting edges do not constitute a vertex, the graph is non-planar. This is often the case for shipping lines and, of course, airline networks. The following graph theoretic measurements are employed; the number of vertices, number of edges, mu  $(\mu)$ , beta  $(\beta)$ , and gamma  $(\gamma)$ . Though numerous other indices exist, the ones employed here are suited for analysis of national airline systems.

Of the graph theoretic measures used the edges and vertices are the most easily understood since they are counted directly from the network diagram (see figure 1). The other indices are algebraically computed using the number of edges and vertices as variables.

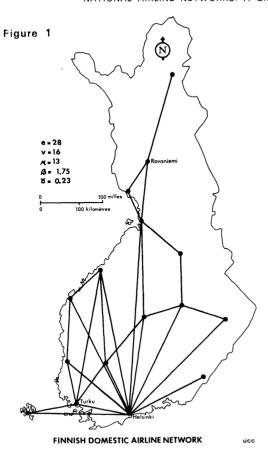
Since the number of independent circuits,  $\mu$ , can also be determined directly from the network map, it is perhaps the most easily understood of the computed indices. This index is computed using the equation:  $\mu = e - v + \rho$ , where e is the number of edges or routes, v is the number of vertices, and  $\rho$  is the number of non-connected subgraphs. If no circuits (loops) exist in a graph,  $\mu$  will equal zero. The higher the  $\mu$  value the more interconnected the graph. It is here hypothesized that the economically more developed countries have highly interconnected airline networks.

The beta index  $(\beta)$  indicates the relationship between the two basic elements of a network and is expressed as  $\beta=\frac{e}{v}$ . Transportation networks with complicated structures have high values of  $\beta$ , simple networks have low  $\beta$  values. Although values less than one can occur for disconnected graphs, they are uncommon since a value of one is ascribed to any network which has only one circuit. The beta increases with increasing number of circuits. Thus, the range for the non-planar  $\beta$  index can vary from zero to a finite number which is a function of the number of vertices.

The gamma index  $(\gamma)$  is a measure of directness or connectivity, i.e., it expresses the probability of a non-stop connection between any two vertices in the system. It is defined computationally for non-planar graphs as:

$$\gamma = \frac{e}{v(v-1)}$$

<sup>&</sup>lt;sup>2</sup> See for example Chorley and Hagget (1970).



The index is a ratio between the observed number of edges (routes) and the maximum number of egdes obtained by interconnecting all of the vertices. For completely interconnected networks  $\gamma=1.0$ , and it approaches zero as the number of edges decreases or the number of vertices increases.

The graph theory indices described above constitute the core of the analysis (Table 1), since in effect they represent mathematical rules for transforming specific features of the transport system into a set of numerical values. This quantitative process, being the logical basis for the analysis, directly determines the conclusions of this study. In a later section of this study, the validity of such a mathematical procedure is evaluated.

### **HYPOTHESES**

It is hypothesized that the graph theoretic indices relating to complexity  $(\beta)$ , number of circuits  $(\mu)$ , connectivity  $(\gamma)$ , number of edges (e), and number of vertices (v) can be explained by national, economic, demographic, topographic and political characteristicts. These causal characteristics include the level of economic development, topographic relief, size in area, total population, and population distribution. All of the graph theoretic indices, as dependent variables, describe distinct facets of the networks, and cannot be explained in a regression analysis by only one set of causal factors. It is hypothesized, however, that each of the previously stated independent variables is sufficiently important for inclusion in each regression equation even though the causal links are distinctly different in each equation.

With respect to specific indices it is hypothesized that the number of edges and vertices will be primarily a function of population, area, level of economic development and to a lesser extent topographic relief. These independent variables are also asserted to be causal determinants of  $\mu$  since it is largely dependent upon the number of edges. Beta, a measure of network

complexity, will correlate directly with the level of economic development, and population. The relationship between beta and area will either be insignificant or be inverse, because as the area increases the number of isolated spurs to remote areas increases, thereby reducing the overall interconnectedness. Lastly, the gamma index will be high where the system only serves the major cities, but not the less populated settlements. If a country has a large number of small centers with air connections it is likely these centers will be connected with only major settlements, resulting in a small gamma.

In these equations surrogate measures for the level of economic development include per capita income, percent of the labor force in primary activities, per capita value of foreign trade, and per capita energy consumption. The relief variable is based on a series of cross-sectional profiles traversing the country. (Kansky, 1963, pp. 45-46). Per cent urban describes the population distribution. Total population and area are self explanatory. These eight variables are included as the independent variables in the subsequent regression analyses.<sup>3</sup>

### REGRESSION ANALYSES

Step-wise multiple regression analysis reveals that of the eight hypothesized variables, population, per capita income, and area are significant causal variables in four of five equations (see table 2). Relief and per cent urban, early to enter the equations on several occasions, are each statistically significant only once. Per cent of the labor force in primary activities and per capita energy consumption, indicators of economic development, are not significant in any of the equations.

With regard to specific relationships, it appears from Table 2 that the number of edges and vertices are highly correlated since the same independent variables constitute the regression equations. Indeed the Pearsonian correlation coefficient between edges and vertices is 0.95. The same independent variables (income, population, area) are statistically significant in explaining the number of circuits.

Beta, exhibiting the complexity of the systems, was primarily a function of the population and per capita value of foreign trade. This suggests that large populations with high levels of economic development as measured by international trade demand complex networks. Conversely, many of these indicators exert just the reverse effect on the gamma index. The size of the country, income, and relief are negatively correlated with gamma, verifying

<sup>&</sup>lt;sup>3</sup> Rather than using several variables it may have been theoretically preferable to use factor scores from factor analysis. Such factor scores were not readily available. Moreover, factor scores are not necessarily a better indicator of economic development than per capita income. In fact, factor scores contribute to a higher level of abstraction not desired in this study.

Table 1

Graph Theoretic Measures

Country	Edges e	Vertices v	Mu μ	Beta $eta$	Gamma γ
Algeria	24	16	9	1.50	0.20
Angola	56	27	30	2.07	0.16
Bolivia	46	31	16	1.48	0.10
Bulgaria	11	11	1	1.00	0.20
Ceylon (Sri Lanka)	6	5	2	1.20	0.60
Chile	50	31	20	1.61	0.11
Cuba	12	9	4	1.33	0.33
Czechoslavakia	21	10	12	2.10,	0.47
Finland	28	16	13	1.75	0.23
France	108	49	60	2.20	0.09
Ghana	5	4	2	1.25	0.83
Hungary	0	0	0		
Iran	36	17	20	2.12	0.26
lraq	2	3	0	0.67	0.67
Malaysia	16	9	8	1.78	0.44
Mexico	100	60	41	1.67	0.06
Nigeria	31	12	20	2.58	0.47
Poland	14	8	7	1.75	0.50
Romania	16	12	5	1.33	0.24
Sudan	33	18	16	1.83	0.22
Sweden	69	24	46	2.88	0.25
Thailand	50	20	31	2.50	0.26
Tunisia	1	2	0	0.50	1.00
Turkey	21	15	7	1.40	0.20
Yugoslavia	33	15	19	2.20	0.31

Table 2 Regression Equations Sequence of Entry in Stepwise Multiple Regression

Dependant variable  1. edges (e)			Indepe	ndant		r <sup>2</sup>		
	=	- 7.83	+ 0.000576 pop. (.000244)	+ 0.0217 income (.0059)	+ 0.0172 area (.0058)	+ 0.215 relief * (.325)	061	
2.	vertices (v)	=	- 2.53	+ 0.000218 pop. (.000105)	+ 0.00995 area (.00301)	+ 0.00810 income (.00304)	+ 0.266 relief (.166)	0.59
3.	mu $(\mu)$		- 1.46	+ 0.0156 income .00409)	+ 0.000369 pop. (.000124)	+ 0.00704 area (.00319)	- 0.0123 urban * (.0155)	0.62
4.	beta <i>(β)</i>	=	1.37	+ 0.0000289 pop. (.00000637)	+ .00375 trade (.00107)	<ul><li>0.00208 urban</li><li>(.000712)</li></ul>	- 0.000682 income * (.000374)	0.57
5.	gamma $(\gamma)$	=	0.761	- 0.000149 area (.0000605)	- 0.000196 income (.0000788)	- 0.00723 relief (.00302)	- 0.000268 labor force * (.000295)	0.46

<sup>\*</sup> Not significant at the 0.05 level.

Numbers in parentheses are the respective standard errors.

Variable units: population in thousands

area in thousands of square kilometers income: per capita in US Dollars trade: " "

urban: percent relief: an index

labor force : percent

the hypothesized relationship. As these causal variables increase in magnitude so do the number of vertices and edges, as per equations 1 and 2 (table 2). However, while the number of edges increases arithmetically, the potential number of edges increases geometrically. Therefore, since France has so many vertices (108) and since numerous potential non-stops links are not yet connected, it has a low gamma value (0.09, see table 1), yet the network is exceedingly complex as demonstrated by a high beta index (2.2).

# **RESIDUALS**

The list of residuals (table 3) reveals that the hypothesized explanatory variables were least appropriate for Angola, France, Mexico, Yugoslavia, Ghana, and Hungary. Hungary, with no domestic network was the only country with consistently negative residuals. In fact it skewed the residuals so as to prelude the possibility of other large negative residuals. Angola, to the contrary, has a network which far exceeded the expected number of edges, vertices, and circuits. Although the rail system is adequate, the Angola highway system is restricted, emphasizing the need for air connections though not necessarily frequent service. Ghana's and Tunisia's high positive residuals for gamma are attributable to the extreme simplicity of the networks. With only five edges and four vertices, Ghana is quite well connected and scores high on the gamma index. France and Mexico exhibit high residuals for the number of edges and vertices. Since the networks are too extensive to be completely interconnected the gamma indices are rather low. Yugoslavia's airline network, catering in great part to tourist travel, exhibits the largest positive beta residual.

# **EVALUATION**

The analysis of national airline networks has focused on easily quantifiable characteristics of the countries as well as the basic topological properties of the networks. The coefficient of determination attest to the fact that numerous pertinent causal factors remain unidentified. The number of vertices, for example, is a product of intercorrelated factors ranging from the previously discussed to those more difficult to assess, such as national policy and prestige associated with extensive airline service, significance of alternative modes and the nature of the settlement pattern. In the latter case it is probable, for example, that every urban center in a developed country with a population of over 100,000 has commercial airline service, provided it is not in close proximity to a large urban area. Therefore the number and distribution of such centers is vital to the network properties. However, for smaller cities factors such as (1) proximity to the closest airline service (remoteness), (2) severity of climate, (3) potential for year-round surface transportation alternatives, and (4) the specialized function of the city may be important in determining whether commercial services exist and in turn the value of graph theoretic measures.

Table 3

List of Residuals Stepwise Multiple Regression

Edges	Vertices	Mu	Beta	Gamma
Residual	Residual	Residual	Residual	Residual
-20.4295	-10.4174	-10.1299	-0.1154	-0.0133
36.3965	17.2679	19.3680	0.2014	-0.2958
22.3029	12.0279	10.1398	0.5206	-0.1324
4.0318	5.4876	-2.4896	0.7524	-0.2695
1.9726	3.3665	-1.2218	-0.0883	-0.0020
-15.9129	-8.5087	-8.1170	-0.3243	0.0881
8.6029	4.0489	3.8788	0.3717	-0.2763
4.7454	1.6925	4.8759	0.2818	0.1630
-13.9954	-0.3246	-13.6040	-0.1560	-0.0326
30.8209	14.7413	17.6753	0.3517	-0.0338
-4.4523	-1.7886	-2.4286	0.1835	0.3202
-30.9864	-14.9353	15.1931	0.9486	-0.2933
-5.8602	-6.4944	0.6405	0.4537	-0.0502
-8.2077 	-5.2406	-3.9311	-0.3952 	0.1239
2.9222	3.0227	-0.1018	-0.1145	0.0429
29.7343	20.0560	9.6715	-0.1577	-0.0130
-19.4567	-8.5840	-11.8841	-0.3732	0.0241
-9.5934	-4.7593	-3.8008	0.2019	0.0830
-10.0399	-4.7862	-4.1127	0.0953	0.0213
16.6902	-8.9878	6.9611	-0.3171	0.1583
2.7972	-3.1754	4.0506	0.2674	0.1266
21.0074	5.2794	14.6876	0.5847	-0.1800
-1.4886	-2.7195	0.8093	-0.2504	0.4168
-18. <b>7</b> 0 <b>7</b> 0	-8.0855	-10.7610	-0.1633	-0.0549
	Residual  -20.4295 36.3965  22.3029 4.0318  1.9726 -15.9129  8.6029 4.7454  -13.9954 30.8209  -4.4523 -30.9864  -5.8602 -8.2077  2.9222 29.7343  -19.4567 -9.5934  -10.0399 -16.6902  2.7972 21.0074  -1.4886	Residual         Residual           -20.4295         -10.4174           36.3965         17.2679           22.3029         12.0279           4.0318         5.4876           1.9726         3.3665           -15.9129         -8.5087           8.6029         4.0489           4.7454         1.6925           -13.9954         -0.3246           30.8209         14.7413           -4.4523         -1.7886           -30.9864         -14.9353           -5.8602         -6.4944           -8.2077         -5.2406           2.9222         3.0227           29.7343         20.0560           -19.4567         -8.5840           -9.5934         -4.7593           -10.0399         -4.7862           -16.6902         -8.9878           2.7972         -3.1754           21.0074         5.2794           -1.4886         -2.7195	Residual         Residual         Residual           -20.4295         -10.4174         -10.1299           36.3965         17.2679         19.3680           22.3029         12.0279         10.1398           4.0318         5.4876         -2.4896           1.9726         3.3665         -1.2218           -15.9129         -8.5087         -8.1170           8.6029         4.0489         3.8788           4.7454         1.6925         4.8759           -13.9954         -0.3246         -13.6040           30.8209         14.7413         17.6753           -4.4523         -1.7886         -2.4286           -30.9864         -14.9353         -15.1931           -5.8602         -6.4944         0.6405           -8.2077         -5.2406         -3.9311           2.9222         3.0227         -0.1018           29.7343         20.0560         9.6715           -19.4567         -8.5840         -11.8841           -9.5934         -4.7593         -3.8008           -10.0399         -4.7862         -4.1127           -16.6902         -8.9878         -6.9611           2.7972         -3.1754 <t< td=""><td>Residual         Residual         Residual         Residual           -20.4295         -10.4174         -10.1299         -0.1154           36.3965         17.2679         19.3680         0.2014           22.3029         12.0279         10.1398         0.5206           4.0318         5.4876         -2.4896         -0.7524           1.9726         3.3665         -1.2218         -0.0883           -15.9129         -8.5087         -8.1170         -0.3243           8.6029         4.0489         3.8788         0.3717           4.7454         1.6925         4.8759         0.2818           -13.9954         -0.3246         -13.6040         -0.1560           30.8209         14.7413         17.6753         0.3517           -4.4523         -1.7886         -2.4286         0.1835           -30.9864         -14.9353         -15.1931         -0.9486           -5.8602         -6.4944         0.6405         0.4537           -8.2077         -5.2406         -3.9311         -0.3952           2.9222         3.0227         -0.1018         -0.1145           29.7343         20.0560         9.6715         -0.1577           -19.4567</td></t<>	Residual         Residual         Residual         Residual           -20.4295         -10.4174         -10.1299         -0.1154           36.3965         17.2679         19.3680         0.2014           22.3029         12.0279         10.1398         0.5206           4.0318         5.4876         -2.4896         -0.7524           1.9726         3.3665         -1.2218         -0.0883           -15.9129         -8.5087         -8.1170         -0.3243           8.6029         4.0489         3.8788         0.3717           4.7454         1.6925         4.8759         0.2818           -13.9954         -0.3246         -13.6040         -0.1560           30.8209         14.7413         17.6753         0.3517           -4.4523         -1.7886         -2.4286         0.1835           -30.9864         -14.9353         -15.1931         -0.9486           -5.8602         -6.4944         0.6405         0.4537           -8.2077         -5.2406         -3.9311         -0.3952           2.9222         3.0227         -0.1018         -0.1145           29.7343         20.0560         9.6715         -0.1577           -19.4567

Although one of the other weaknesses of this analysis is the general disregard for capacities and numbers of passengers the  $\gamma$  and  $\mu$  values are at least indirectly based on the levels of demand between vertices. If the demand between two points is low and a direct but not non-stop route already exists, then there is little incentive for providing a non-stop link. Moreover, if movement from a city is rather channelized, then there is little potential for interconnection. For example, Acapulco is a specialized resort that has connections with only a few major cities of Mexico, but lacks the multitude of connections common among other cities with lower passenger volumes. Such channelized networks cannot be predicted using the independent variables employed here.

Also the findings are based on twenty-five countries but it is suggested that they apply universally. The selection of countries, however, may not be ideal for supporting such an inductive assertion. Since the countries were sampled in the early 1960's, recently formed nations, particularly in Africa, are excluded from consideration. Nevertheless, these new countries may truly be unique cases and would not enhance the analysis. But one could still criticize the selection of countries for over-representing Europe and socialist countries. Ironically the two Latin American countries recently with socialist governments, Cuba and Chile, are included in the sample. For the sake of comparison with Kansky's study and to avoid the recently formed small underdeveloped nations, the sample used here has distinct advantages.

There are two fundamental weaknesses of graph theory which have not been sufficiently emphasized: the inadequacies of topological measures and difficulty of incorporating passenger levels into the analysis. Clearly, graph theory indices provide a procedure for measuring the topological properties of a network; however, two networks with rather different lengths and shapes may be deemed similar by graph theory. As an illustration, two networks with four nodes and four vertices each will by definition have identical beta, gamma and mu indices, yet they may be quite different in form. They may also carry different volumes of traffic, undetected by graph theory measures. These difficulties must be considered in interpreting the findings.

Lastly, the high degree of interrelatedness among the dependent variables, e, v,  $\beta$ ,  $\mu$ ,  $\gamma$  must be recognised. The high correlations among the dependent variables are implicit from the computational forms defining the graph theoretic measures and can be further elucidated by a path analytic model. The path analytic approach as shown by Pyle and Rees (1971) and Sööt (1969) can be an invaluable tool in interpreting the interrelationships and assessing the indirect effects in a causal model. For instance, the regression analysis suggests that the four independent variables of table 2 explain 62% of the variation in  $\mu$ . We know that the number of circuits in a network is highly dependent upon the number of edges (table 4). Indeed, when the number of vertices is fixed, each additional e corresponds to a new circuit, until  $\alpha$  (the ratio of the number of circuits to the maximum number) is one. How-

ever, the effect through e and v between the dependent variables in the regression analysis and the  $\mu$  value not apparent in regression analysis can readily be seen in figure 2. This path model illustrates that the indirect effects from the four independent variables are all stronger than the direct effects. For example, the bulk of the 0.474 correlation between  $\mu$  and the population levels is accounted for by the intermediate effects of e and v. This can be shown computationally by:

$$\begin{array}{l} r_{37} = p_{37} + r_{17}p_{31} + r_{97}p_{39} + r_{127}p_{3712} \\ .474 = .049 + .473 - .005 - .015 \\ .474 \cong .498 \end{array}$$

The second term on the right side of the equality is an aggregate of all the indirect effects through e and v, and in this instance, quite by chance, the magnitude is almost equivalent to the correlation coefficient. With regard to the direct effects, aside from e, only per capita income has a substantial positive direct relationship.

Figure 2

# SAMPLE PATH ANALYTIC MODEL OF NATIONAL AIRLINE NETWORKS

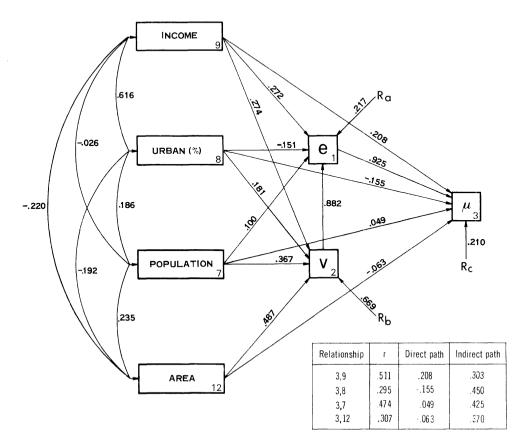


Table 4

Correlations Among Graph Theoretic Measures and Selected Inpedendent Variables

		1	2	3	4	5	6	7	8	9	10	11	12	
e	1	1.000	.953*	.962*	.603*	55 <b>6</b> *	.231	.512*	.324	.416*	.248	090	.405*	1
v	2		1.000	.835*	.439*	598*	.348	.508	.325	.269	.095	042	.478*	2
$\mu$	3			1.000	.708*	469*	.110	.474*	.295	.511*	.195	123	.307	3
β	4				1.000	226	.030	.395*	021	.214	.264	.130	.240	4
γ	5					1.000	333	263	232	301	192	.038	<b>387</b> *	5
Relief	6						1.000	.426*	.367*	020	054	<b>177</b>	050	6
Population	7							1.000	.186	026	232	.161	.235	7
% Urban	8								1.000	.616*	.589*	802*	192	8
Income	9									1.000	.906*	656 <b>*</b>	<b>−.200</b>	9
Foreign Trade	10										1.000	6 <b>78</b> *	312	10
Labor Foreign	11											1.000	.399*	11
Area	12												1.000	12

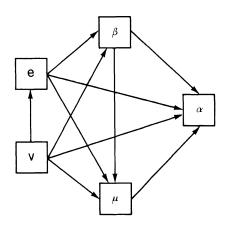
<sup>\*</sup> significant at the 0.05 level.

The same basic interpretation applies to the relationship between edges and the four independent variables. Population, for example, has a relatively weak direct impact on edges, but since it has a sizable impact on the number of vertices, the indirect effect though the latter variable accounts for most of the correlation. This is shown numerically by:

$$\begin{array}{l} r_{17} = p_{17} + p_{27}p_{12} + r_{87}p_{18} + r_{97}p_{19} + r_{12,7}p_{2,12}p_{12} + r_{87}p_{82}p_{12} \\ 0.512 = 0.100 + 0.323 - 0.028 - 0.007 + 0.101 + 0.030 \\ 0.512 = 0.519 \end{array}$$

This illustrates that indirect effects through the number of vertices (variable 2) account for 0.353 (or 0.323 plus 0.030) of the correlation between population and the number of edges.

Figure 3
INTERRELATIONSHIPS AMONG GRAPH THEORETIC MEASURES\*



\* Alpha is computed by dividing the number of circuits in the network by the maximum number of circuits, given the number of vertices. The mathematical expression is:

 $\alpha = -\frac{\mu}{\frac{\nu}{2}(\nu \cdot 1) - (\nu \cdot 1)}$ 

A similar analytic procedure may be applied to a complete system of dependent variables, including at least the twelve variables of table 2. Such a diagram however, would be very complex and would not be required to illustrate the point made by figure 2. Nevertheless, figure 3 is provided to indicate some of the interrelationships which need to be interpreted in evaluating to composition of a measure such as  $\alpha$ . It summarizes some of the interrelationships among the « dependent » variables in the interpretation of alpha, in addition to highlighting the problem of indirect ef-

fects. When figure 2 and 3 are combined (mentally) it is evident that by regressing the four independent variables against measures such as  $\mu$  and  $\alpha$ , much of the actual causal chain leading to the dependent variable remains hidden. The understanding of these causal chains is essential to the effective utilization of graph theory.

# CONCLUSION

As an abstraction of reality, the graph theoretic technique is subject to information loss and more importantly, although it is basically easy to understand, the interpretations can be tricky and are best achieved after first hand experience with the method. Without doubt, the  $\mu$ , for example,

expresses the number of circuits in a network, yet one might question the utility of knowing this fact. After comparing networks and their  $\mu$  values, it becomes apparent that the measure is indeed meaningful. Also it must be acknowledged that this variable only measures one characteristic of the network and a series of indices must be utilized to adequately describe the wide range of characteristics displayed by a network. As such, only in the aggregate do graph theoretic indices adequately measure the multitude of features displayed by a network. Indeed the problem of comparing networks on the basis of complexity and connectivity is a task quite easily performed with graph theoretic indices.

In the main the study has attained its principal objective of identifying the interrelationships among airline networks and a few economic, topographic, and geographic characteristics of nations. Specifically it has established that the level of economic development, relief, size of country, population, and urbanization are all important determinants of network structure. More generally graph theory is found to be an invaluable tool, despite its drawbacks, for network analysis and its use should be encouraged.

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## **ABSTRACT**

# SOOT, Siim: National airline networks: a graph theoretic analysis

Network properties of national domestic airline systems are examined and linked to causal factors such as levels of economic development, population size and distribution, topographic relief, and size of country. Graph theoretic indices are utilized to measure network characteristics and become the independent variables in regression analyses. The theoretical pitfalls of this method are highlighted by utilizing a path analytic framework to identify the degree of interrelationship among the dependent variables. Still, the graph theoretic method is deemed useful as a means of topologic analysis of network structures.

KEY WORDS: Transportation geography, national airline networks, graph theory, topologic analysis

#### RÉSUMÉ

SOOT, Siim : Les réseaux aériens nationaux : une analyse basée sur la théorie des graphes

Les propriétés des réseaux des systèmes aériens domestiques de différentes nations sont étudiées et mises en relation avec un certain nombre de facteurs tels que les niveaux de développement économique, l'importance et la répartition de la population, le relief et la dimension du pays. Les indices de la théorie des graphes sont utilisés pour mesurer les caractéristiques des réseaux et deviennent les variables dépendantes dans les analyses de régression. Les écueils théoriques de cette méthode sont mis en évidence lorsqu'on utilise une analyse de cheminement pour identifier le degré d'interrelation entre les variables dépendantes. Cependant, l'utilisation de la théorie des graphes s'avère utile comme moyen d'analyse topologique de la structure des réseaux aériens.

MOTS CLÉS: Géographie des transports, réseaux aériens nationaux, théorie des graphes, analyse topologique