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THE METHODOLOGY OF PTOLEMAIC ASTRONOMY: AN ARISTOTELIAN VIEW

Harvey L. MEAD

Philosophy has become mathematics for modern thinkers... yet on this view the whole study of natural science is abolished.

Metaphysics I, 8.

FROM the outset of the long history of inquiry into natural phenomena which marks an important element in the development of Greek philosophy, two divergent, yet ultimately complementary points of view are dominant. The one, frequently associated with the Pythagoreans, tends to rely heavily on mathematical explanations of natural events; the other seeks explanations in natural causes, such as the four elements in the philosophy of Empedocles.[1]

These two Greek traditions reach their point of critical development in the work of Plato and Aristotle, respectively. Plato, in the *Timaeus*,[2] presents his cosmology as a "likely story", incorporating mythical, mathematical, and observational materials into his account. He does not present the work as demonstrative, since in his view we cannot have proper knowledge of the world of becoming and appearance. The recourse to mathematical explanations in the work is a consequence of this view, since, for Plato, mathematics studies objects which are outside the realm of becoming, and which are thus knowable.

Aristotle's series of natural treatises, the *Physics*, the *De Caelo*, the *De Generatione et Corruptione*, and the *De Anima* with the subsequent biological treatises, attempts to present an account of natural phenomena in their own terms, via principles with the same conceptual bases as the material, sensible world which they explain. At the same time, the treatises constitute Aristotle's response to the Platonic account; a sign of this is that, generally speaking, the treatises, ordered in this way,[3] follow the outline of the *Timaeus*.

1. Among a large number of histories, cf. J. BURNET, *Early Greek Philosophy*, 4th ed. (London, Black, 1930). One account of the role of the Pythagoreans can be found in W. A. HEIDEL, "The Pythagoreans and Greek Mathematics," *American Journal of Philology*, 61 (1940).
2. The question concerning the Pythagorean elements in the *Timaeus* forms a part of most discussions of the work. For a modern debate on the subject, cf. A. E. TAYLOR, *A Commentary on Plato's Timaeus*, (Oxford University Press, 1928), and F. M. CORNFORD, *Plato's Cosmology: The Timaeus of Plato with a running commentary*, (New York, Library of Liberal Arts Press, 1957).
3. This is the order proposed by Aquinas; cf. *In I Meteor.*, lect. 1, n. 3, and *In De Generatione et Corruptione*, prœmium, n. 1.

The *De Caelo*, in fact, is responding to more than the *Timaeus*. Behind the general remarks of Plato in that work lies a treatise which has not survived.[4] It was an essay on mathematical astronomy which, according to Simplicius, was commissioned by Plato, and written by the Greek mathematician Eudoxus. Plato is reported to have instructed Eudoxus to develop a system which would account for the known celestial phenomena, such as the diurnal movement of the heavens, the movement of the sun through the zodiac, and planetary retrogradation, in terms of circular hypotheses, as they are called by the ancient astronomers. The *Timaeus*, then, deals generally with what Eudoxus treats of in detail; Aristotle, in his work, refers to both Plato and Eudoxus as representatives, equally, of the mathematical tradition.

The attempt of Eudoxus starts with the philosophical principle that circular motion is in some sense simple and perfect, and appears to be the motion proper to the celestial sphere. It thus seemed reasonable to extend the application of the circular principle to phenomena in which it did not appear to be operative, as in the case of the planetary motions with their periods of retrogradation. In response, then, to Plato's instruction, Eudoxus developed a system of concentric spheres. All of the spheres needed for the explanation of a single planet's movement are attached to each other, but in different places, such that their rotation is about different poles, at different speeds, as needed to "save the appearances".[5]

The origins of this system are visible in the *Timaeus*. In his *Metaphysics*,[6] Aristotle also discusses the theory of concentric, or homocentric spheres. After noting improvements to Eudoxus' system by Callippus, Aristotle proceeds to add a number of "counteracting" spheres, which permit the spheres proper to each planet to be placed in connection with those relating to the others. In doing so, he attempts to bring the system within a single framework, susceptible, to at least some degree, of a natural, mechanical interpretation. Aristotle alludes to the existence of the theory, further, throughout the *De Caelo*, where he suggests that it represents the work of an alternative, and complementary approach to the study of celestial phenomena.

Both Plato and Aristotle, then, are aware of, and respect, inquiries by mathematicians into natural phenomena. In the case of Plato, strict accuracy is of

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4. L. Schiaparelli, "Le sfere omocentriche di Eudosso, di Callippo, e di Aristotele," *Publicazioni del R. Ossovatoria di Brera in Milano*, ix, (Milan, 1875), has reconstructed, in large part, this ancient system. Cf. also Th. H. Martin, "Mémoires sur les hypothèses astronomiques d'Eudoxe, de Callippe, et d'Aristote," *Mémoires de l'Académie des Inscriptions et Belles Lettres*, xxx, (Paris, 1881) and P. Thirion, "Pour l'astronomie grecque," *Revue des questions scientifiques*, 2e série, t. 15-16 (Louvain, 1898).

5. In fact, Eudoxus appears to have joined only the spheres involved in the explanation of the motion of each planet. This suggests that he did not consider his system as a natural explanation, but rather as a mathematical means of representing the motions of the planets and of calculating with the observations; cf. J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler*, (New York, Dover, 1953), ch. 4. The notion of "saving the appearances", which is associated with the latter interpretation of classical astronomical systems, and which derives from the account of Simplicius mentioned above, forms the starting point for P. Duhem's ΣWZEIN TAΦAINOMENA: *Essai sur la notion de théorie physique de Platon à Galilée*, (Paris, Gallimard, 1908), a work which is an indispensable introduction to the study of the history of the middle science of astronomy.

6. *Metaphysics*, XII, ch. 8. Unless otherwise noted, all references to the works of Aristotle will be to the translations of the Oxford edition, as found in *The Basic Works of Aristotle*, ed. R. McKeon, (New York, Random House, 1941).

little significance, and only the general principles are of concern to him; even his defense of circularity as essential to basic cosmological reasoning remains implicit. For Aristotle, however, a knowledge of all domains is the end of the philosophical enterprise. This includes the natural world, and it is therefore of real concern to him that attempts to account for the movements of the planets — to take an example — be based on proper natural principles, and that they *adequately* account for those phenomena.

Aristotle's emendations of the Eudoxian theory, in the *Metaphysics*, and in his criticism of the entire cosmological structure of Plato, in the *De Caelo*, reveal a clear-headedness about the function of mathematical concerns in the study of nature. That study must guide any subordinate attempt to describe, mathematically, certain natural phenomena. Alternatively, the natural philosopher must maintain close contact with the researches of specialists, whose results are of interest to him as revealing phenomena in need of further explanation.

After Aristotle, however, the sense of complementarity between mathematical and natural investigations of the same phenomena is soon lost. The two traditions widespread in Greek philosophy, and temporarily recognized in the thought of Aristotle, again diverge, as the enormous influence of Plato and Aristotle on philosophers in succeeding centuries leads to their continuation. Within the tradition stemming from the Pythagoreans and Plato, little effort seems to be made towards the development of a scientific methodology which explains and justifies the widespread use of mathematics in the consideration of natural phenomena. Philosophical inquiry, on the Platonic model, gives way to the research into various phenomena susceptible of mathematical description.[7] In large part, this neglect is a concomitant to the Platonic view already alluded to, and expressed at the outset of the *Timaeus*, that the material world of appearance is not scientifically understandable, while mathematics belongs to the realm of being and true speculative thought.

On the other hand, the commentators on Aristotle's works do little more than translate his many references to the various specialized fields of inquiry which are the forerunners of mathematical physics. Few are in contact with the ongoing research in such areas as astronomy, optics, mechanics, and harmonics. As a result, the sense of complementarity so clear in Aristotle's writing is gradually lost.

During this period, the major contributions to the growing understanding of celestial phenomena are made by specialists working in mathematical astronomy. It is here that more and more accurate observations are accumulated, and an ingenious system for dealing with them devised by distinguished mathematicians. Ptolemy, whose work in astronomy represents the culmination of six centuries of ancient inquiry, characterizes the point of view of many classical astronomers at the outset of his major treatise, the *Almagest*:

> And therefore, meditating that the other two genera of the theoretical would be expounded in terms of conjecture rather than in terms of scientific understand-

7. This is not to deny that Ptolemy and other ancient astronomers were in possession of a powerful methodology. Cf. L. O. KATTSOFF, "Ptolemy and Scientific Method," *Isis*, 38 (1947), and O. NEUGEBAUER, "The History of Ancient Astronomy; Problems and Methods," *Publications of the Astronomical Society of the Pacific*, 58 (1946).

> ing: the theological because it is in no way phenomenal and attainable, but the physical because its matter is unstable and obscure, so that for this reason philosophers could never hope to agree on them; meditating that only the mathematical, if approached enquiringly, would give its practitioners certain and trustworthy knowledge with demonstration both arithmetic and geometric resulting from indisputable procedures, we were led to cultivate this discipline.[8]

While Ptolemy feels that this division is Aristotelian, his approach is clearly Platonic, in its view of the value of the "physical genus" of theoretical inquiry.

In his work, he will for the most part lean to the requirements imposed by his intention of describing the mathematical aspects of the celestial phenomena; fundamental inquiry into philosophical principles are not very significant in his major treatise. Ultimately, however, his work has its foundations in Aristotle's arguments, in the *Physics* and the *De Caelo*, according to which the movement proper to aithereal (i.e. heavenly, in contrast with sub-lunar) bodies is circular. The ongoing work of men like him will eventually lead to a rejection of the Aristotelian position, when Kepler discovers that the orbits of the planets are elliptical. In the seventeenth century, a need for a new set of cosmological principles is a direct consequence of that discovery.

Ptolemy's rejection of the attempts to understand natural phenomena in their own terms has its basis in a fact which was not necessarily fully clear to him. Due to a lack of accurate data concerning celestial phenomena, the ancient astronomer was simply unable, in important instances, to pursue the study of their causes. Aristotle had, in the minds of many, succeeded in presenting a satisfactory account of the main principles, according to which the principle of natural place guides the entire study of the spatial movement of bodies. Those bodies are divided into aithereal and sub-lunar; and the proper movement of aithereal bodies is circular, that of sub-lunar bodies rectilinear. But these principles were exceedingly general, and their proper application required a descent to more particular, and more specific, fields of inquiry.

Investigators such as Ptolemy, then, are seeking a mathematical description of various natural phenomena. At least temporarily, they are almost obliged to prescind from attempts to explain them. The major emphasis in their enterprise is on the construction of a system of propositions which at least permits the manipulation and organization of the masses of observational material already possessed, on « indisputable procedures » which give « certain and trustworthy knowledge » once the premisses are accepted.

Ptolemy's account of the planetary movements makes use of the hypotheses of the eccentric and the epicycle as such premisses. He enunciates the principle which guides the formulation of these hypotheses, prior to his first employment of them in the treatise on the sun.

> In general the motions of the planets in the direction contrary to the movement of the heavens are all regular and circular by nature, like the movement of the universe in the other direction. That is, the straight lines, conceived as revolving the stars on their circles, cut off in equal times on absolutely all circumferences equal angles at the centres of each; and their apparent irregularities result from

8. PTOLEMY, *Almagest*, tr. R. C. Taliaferro, in *Great Books of the Western World*, ed. R. M. Hutchins (Chicago, Britannica, 1952), vol. 16, I, ch. 1, pp. 5-6.

> the positions and arrangements of the circles on their spheres through which they produce these movements.[9]

As is clear, this is a principle (a "law" in Newtonian terms) of regular circular motion, but it prescinds from a delimitation of the center defining that motion. In the *Almagest*, this need not be the earth. The principle thus marks a departure from the theory of concentric spheres devised by Eudoxus. In this latter case, all the circles do, in fact, center in the earth, and Aristotle, with his geocentric theory of natural place, is thus able to consider the theory a serious effort to account for the phenomena.

In the eccentric hypothesis, the planet is conceived of as fixed in a sphere, rotating about an axis; the center of the sphere is removed somewhat from the center of the universe, the earth. In the epicyclic hypothesis, the star is again fixed in a sphere, but the center of this sphere is attached to the outer surface of another, larger sphere; the center of this second sphere can either be in the earth or eccentric to it. In the latter case, the two hypotheses are in fact joined in a rather more complex one, used by Ptolemy in his account of all the planets except the sun.

Throughout his account, Ptolemy, in his use of these hypotheses, makes use of cross-sections of the spheres through their centers, i.e. he deals with their equators, on which all the planets, and the epicycles' centers, are conceived as fixed. He disregards the spheres themselves. One good reason for this disregard, apart from simplicity of conceptualization, lies in the fact that in the epicyclic theory at least, any imagined spheres would, in their movement, intersect one another. Such an event would create a problem of interpretation, from a natural point of view. Nowhere in his treatise, however, does Ptolemy address himself to the problem of the physical interpretation of his spheres.

For several hundred years prior to his time, in fact, astronomers were so involved with the specialized problems of their discipline that the physical connotations of their hypotheses gradually became irrelevant. As Dreyer comments, "Aristarchus (fl. 280 B.C.) is the last prominent astronomer of the Greek world who seriously attempted to find the physically true system of the world." [10] By the time of Ptolemy, in the second century A.D., the major astronomer of the classical world is able to leave such questions to other inquirers.

This situation continues through the Middle Ages. In the thirteenth century, this division of inquiry between mathematicians seeking to describe natural phenomena, and natural philosophers seeking to give accounts of their causes, is accepted almost without question. Nevertheless, the work of such men as Theodoric of Freiburg and Albertus Magnus indicates that a return to the earlier, comprehensive viewpoint of Aristotle is still possible; Albert's student Aquinas seems more than ordinarily aware of the situation. In the following pages, we shall make use of the insights of this Aristotelian philosopher as a guide for our analysis of the distinguishing characteristics of Ptolemaic astronomy.

Fundamental to these is its hypothetical character, which takes on new

9. *Ibid.*, III, ch. 3, p. 86.

10. Carl DREYER, *A History of Astronomy from Thales to Kepler*, 2nd ed., (New York, Dover, 1959), p. 149.

importance when one becomes aware of the possibility of working with the two traditional areas of research as complementary. Aquinas distinguishes between the doctrinal, and the hypothetical, explanations of natural phenomena in a passage early on in the *Summa.*

> Dicendum est quod ad aliquam rem dupliciter inducitur ratio. Uno modo, ad probandum sufficienter aliquam radicem; sicut in scientia naturali inducitur ratio sufficiens ad probandum quod motus caeli semper sit uniformis velocitatis. Alio modo, inducitur ratio, non quae sufficienter probat radicem, sed quae radici jam positae ostendat congruere consequentes effectus; sicut in astrologia ponitur ratio excentricorum et epicyclorum ex hoc quod, hac positione facta, possunt salvari apparentia circa motus caelestes; non tamen ratio haec est sufficienter probans, quia etiam forte alia positione facta salvari possent.[11]

In contrast with the doctrinal investigations of the *Physics* and the *De Caelo*, the explanatory principles of Ptolemaic astronomy, the epicycle and the eccentric, are not considered true causal accounts of the movements of the heavenly bodies.

Ptolemy, at least, seems to make this point of view almost explicit when he discusses the identity of the two explanations of the sun's movement, in the third book of his treatise. It is perfectly clear that the identity is only one in the mind of someone for whom such details as the actual distances of the planets from the earth is beyond consideration. With the latter element considered, the eccentric and epicyclic hypotheses provide radically different accounts of the data otherwise interpreted only in perspective.

Rather, these hypotheses, in the tradition of "saving the appearances," serve to give the mind some general view of the data, which in their otherwise unsynthesized state defy the imagination's attempts to deal with them as wholes. To substantiate this remark, it will be necessary to place the middle science of astronomy within the context of the division of the sciences.

MATHEMATICS AND NATURAL SCIENCE

The division of the theoretical sciences into natural science, mathematics, and metaphysics is based on a difference among the objects of speculative thought, as speculative. Certain determinations belong to an object of theoretical science (*speculabilia*) due to the nature of the knowing faculty, and others due to the scientific habit which perfects that faculty. Because the mind which theorizes is immaterial, its objects are likewise immaterial; because science considers necessary things, the objects will likewise be necessary and unchanging. Nevertheless, the mind considers its objects in terms of their varying degrees of abstraction from matter and motion. The different degrees are marked by different modes of defining.

11. Ia, q. 32, a. 1, ad 2; cf. by way of contrast *In II De Caelo*, lect. 15, n. 431, where Aquinas remarks that astronomers *sufficienter* discuss certain phenomena analogous to those referred to in the passage from the *Summa*. In the *De Caelo* passage, they discuss subjects which cannot be considered by natural scientists, due to the element of calculation which they involve; within such a context, he says, they give a sufficient, or specialized, explanation. For similar remarks by Aquinas, cf. *In I De Caelo*, lect. 3, n. 28; *In II De Caelo*, lect. 9, n. 400; lect. 17, n. 451; lect. 18, n. 470; lect. 19, nn. 474–477 (Rome, Marietti, 1954).

> Secundum ordinem remotionis a materia et a motu, scientiae speculativae distinguuntur.
>
> Quaedam igitur sunt speculabilium quae dependent a materia secundum esse, quia non nisi in materia esse possunt, et haec distinguuntur: quia dependent quaedam a materia secundum esse et intellectum, sicut illa in quorum definitione ponitur materia sensibilis; unde sine materia sensibili intelligi non possunt, ut in definitione hominis oportet accipere carnem et ossa, et de his est physica, sive scientia naturalis.
>
> Quaedam vero sunt quae quamvis dependeant a materia secundum esse, non tamen secundum intellectum, quia in eorum definitionibus non ponitur materia sensibilis, ut linea et numerus, et de his est mathematica.
>
> Quaedam vero sunt speculabilia quae non dependent a materia secundum esse, quia sine materia esse possunt, sive numquam sint in materia, sicut Deus et Angelus, sive in quibusdam sint in materia, et in quibusdam non, ut substantia, qualitas, potentia, actus, unum et multa et huiusmodi, de quibus omnibus est theologia.[12]

In this essay, we are concerned with the first two divisions of theoretical science, natural science and mathematics, whose objects *dependent a materia secundum esse*. For the middle sciences are neither wholly mathematical nor wholly natural. The relationships between the two fields, then, are of essential importance in an effort to deal with the middle science of astronomy.

The mathematical sciences demonstrate certain properties of the two species of quantitative subject, the discrete and the continuous. These subjects are abstract wholes composed of (potential and actual) homogeneous and distinct parts. Their consideration does not take into account the many accidents and sensible qualities of the objects of the natural world. The sciences within the domain of geometry thus possess the same mode of definition, and are distinguished, within the inquiries proper to that mode, by a further delimitation of their respective subjects and the appropriate measures of those subjects.

It belongs to natural science, on the other hand, to consider the bodies of the natural world in terms of the sensible properties by which they are known to us in our daily experience. Through his concepts incorporating such properties, the natural scientist deals with the world on its own terms, as it were. While he abstracts from individual characteristics, like any other science, he does not neglect what belongs to the common sensible matter appropriate to the different species of natural object, as such. His definitions contain a reference to the sensible matter which is necessary for a proper knowledge of natural species, and which contributes to their distinction from one another.

Nevertheless, natural science cannot proceed without taking into account the quantitative nature of its subject. A relationship therefore exists between natural science, and mathematics, which studies that quantitative substrate. This relationship is well characterized by Aquinas in the opening paragraph of his commentary on the *De Caelo*. There, developing Aristotle's remark that "the science of nature is for the most part plainly concerned with bodies and magnitudes, and with their changing properties and motions, as also with the principles which belong to that class of substance," he comments:

12. *In librum Boetii De Trinitate Expositio*, lect. 2, q. 1, a. 3.

> De quibus tamen aliter considerat naturalis quam geometra. Naturalis quidem considerat de corporibus inquantum sunt mobilia, de superficiebus autem et lineis inquantum sunt termini corporum mobilium; geometra autem considerat de eis prout sunt quaedam quanta mensurabilia.[13]

Geometry seeks to reach a knowledge of bodies and the relations of their parts in terms of the measures of those bodies; natural science considers those bodies in so far as they are substances in the sensible world of movement and change.

Later in the *De Caelo*, Aquinas develops the implications of these earlier remarks.

> Naturalia autem se habent per appositionem ad mathematica; superaddunt enim mathematicis naturam sensibilem et motum, a quibus mathematica abstrahunt; et sic patet quod *ea quae sunt de ratione mathematicorum salvantur in naturalibus.*[14]

What is discovered about the subjects of geometry and arithmetic holds true of the extended bodies considered in natural philosophy. Even the principle of the infinite divisibility of mathematical bodies is carried over into natural investigations, in that such general inquiries as those found in *Physics VI* do not take into account the natural limitations of any specific *mobile*. Aquinas characterizes the manner in which a transfer of mathematical concepts to the natural domain takes place, in his commentary on the *De Trinitate*, from which we have already quoted above.

> Quanto scientia aliqua abstractiora et simpliciora considerans, tanto eius principia sunt magis applicabilia aliis scientiis. Unde principia mathematica sunt applicabilia naturalibus rebus, non autem e converso, propter quod physica est ex suppositione mathematicae, sed non e converso, ut patet in III *de Caelo et Mundi.*[15]

Because mathematical subjects are known by a process of abstracting from natural objects, they maintain their contact with the latter reality, and their study is applicable, within certain limits, in the natural study of those original objects. A relationship between the two domains is thus seen to hold.

There are, in fact, two uses of mathematics by natural science which are found in the writings of classical philosophers, and which seem to merit the designation "normal".

The first of these uses permits the natural philosopher to apply the geometric study of bodies to his enterprise. This is, in fact, "plainly concerned with bodies and magnitudes," as is clear from numerous passages in the *Physics*, the *De Caelo*, and the *De Generatione*. Nevertheless, these studies involve the investigation of more aspects of the object than are considered by the mathematician; these are the sensible properties of those objects as they exist *in rerum natura*. For example, in considering eclipses in the *De Caelo*, Aristotle makes use of the geometry of circles and spheres, in such a way that the properties demonstrated by the mathematician are shown to apply

13. *In I De Caelo*, lect. 1, n. 7.

14. *In III De Caelo*, lect. 3, n. 568 (our underlining).

15. *Loc. cit.*

to the natural objects also. Since the discussion never loses sight of the fact that these are natural, and move, the point of view is different from that of the mathematician.

A second use of mathematics occurs in the attempts, in the *Physics* and the *De Caelo* primarily, to deal with the motion of bodies through space. In books six and eight of the *Physics*, for example, Aristotle deals with such subjects explicitly, and proceeds on the assumption of the geometry of two and three dimensions.

In both cases, the entities discussed by the mathematician are understood differently by the natural philosopher, who is looking at them as, or in their relation to, natural bodies undergoing change of some kind. Ultimately, these two uses of mathematics come together in the study of natural objects. It is the role of the natural philosopher to deal with their changes, fundamental to all of which is change of place, while the proper discussion of the movement of bodies according to place is dependent upon an understanding of the bodies themselves.

A third, different use of mathematics — not obviously "normal" — appears to occur in the middle sciences. As already noted, data were frequently unavailable in many areas; the position of the different parts of the rainbow, the actual orbits of the planets, and the relationships of the sounds produced by different strings of different sizes all needed careful investigation, and were not fully determined.

The attempt to obtain these data — the major enterprise of at least the middle science of astronomy — involves a specialized use of mathematics having its foundation in the normal uses outlined above, but introducing new elements, and much more detaiied procedures. Further, these classical sciences apparently operate within the two modes of defining which distinguish the investigations of mathematics and natural science. The two aspects of the inquiry, in the case of astronomy, are not easily separated, particularly in the part of the science which is most significant, where the mathematical hypotheses are brought to bear on the data in need of organization.

The general name of this group of middle sciences (*scientiae mediae*) in fact derives from a hesitancy as to their proper place in the scheme of the sciences. The manner in which these researches, particularly those concerned with celestial phenomena, relate to the Aristotelian tradition expounding certain non-hypothetical natural principles also remains unclear. We shall attempt in the final section of this essay to deal with these problems, with the ultimate intention of placing the middle science of astronomy in a proper context.

THE METHODOLOGY OF PTOLEMAIC ASTRONOMY

Fundamental to the work of the middle science is a heavy reliance on mathematical techniques and principles, coupled with considerable observational detail. The general name of this group of sciences in fact derives from a hesitancy as to their proper place in the scheme of the sciences. That they do not belong, strictly speaking, in either of the two broad classes of mathematics and natural science is pointed out by Aquinas in the following passage of the *De Trinitate*:

> Et inde est quod de rebus naturalibus et mathematicis ordines scientiarum tres inveniuntur. Quaedam enim sunt pure naturales, quae considerant proprietates

rerum naturalium inquantum huiusmodi, sicut physica et agricultura et huiusmodi.

Quaedam vero sunt pure mathematicae, quae determinant de quantitatibus absolute, ut geometria de magnitudine, arithmetica de numero.

Quaedam vero sunt mediae, quae principia mathematica ad res naturales applicant, ut musica, astrologia, et huiusmodi, quae tamen magis sunt affines mathematicis, quia in earum consideratione id quod est physicum, est quasi materiale; quod autem mathematicum, quasi formale; sicut musica considerat sonos non inquantum sunt soni, sed inquantum sunt secundum numeros proportionabiles, et sic est in aliis. Et propter hoc demonstrant conclusiones suas circa res naturales, sed per media mathematica; et ideo nihil prohibet si inquantum cum naturali communicant, materiam sensibilem respiciunt. Inquantum enim cum mathematica communicant, abstractae sunt.[16]

These sciences fall somewhere in between, in that they apply mathematical principles to natural objects; thereby, Aquinas suggests, they have something in common with both of the classes of theoretical science. Because they have mathematical principles of demonstration, however, they are here classed with the mathematical sciences, more than with the natural.

The natural objects with which these sciences are concerned, however, turn out to be those possessing certain mathematical properties. In the case of astronomy, for example, the consideration of the orbits is based on the natural divisibility of the space traversed. Generally speaking, a study of such a subject would belong to that of the natural scientist. In the *Physics*, Aquinas affirms this view.

Dicuntur autem scientiae mediae, quae accipiunt principia abstracta a scientiis pure mathematicis, et applicant ad materiam sensibilem; sicut *perspectiva* applicat ad lineam visualem ea quae demonstrantur a geometria circa lineam abstractam; et harmonica, idest *musica*, applicat ad sonos ea quae arithmeticus considerat circa proportiones numerorum; et *astrologia* considerationem geometriae et arithmeticae applicat ad caelum et ad partes eius. Huiusmodi autem scientiae, licet sint mediae inter scientiam naturalem et mathematicam, tamen dicuntur hic a Philosopho esse magis naturales quam mathematicae, quia unumquodque denominatur et speciem habet a termino; unde, quia harum scientiarum consideratio terminatur ad materiam naturalem, licet per principia mathematica procedant, magis sunt naturales quam mathematicae.[17]

In these remarks, we have Aquinas' most considered judgement on the middle sciences, or so it appears to this writer. In spite of numerous comments to the effect that these areas of research are either mathematical, or "middle", one must ultimately class them among the natural sciences. For the object of a science is its conclusions, and the conclusions of the middle sciences deal with natural phenomena.

The mathematics used in astronomy, then, is a special tool of the investigator, who must ultimately be considered a natural scientist. It does not, however, appear to enter into the investigations of that science in either of the "normal" ways outlined above. Indeed, the *Almagest* — Ptolemy's second century treatise on astronomy — reveals to the reader a complexity in the development of its argument that calls for a methodological analysis, if we are to grasp its implications.

16. *Ibid.*

17. *In II Physic*, lect. 3, n. 164.

The Ptolemaic approach is a development of the Eudoxian theory of concentric spheres. Aristotle's earlier emendation of that theory, in the twelfth book of the *Metaphysics*, is the locus for careful analysis by many of his commentators. Aquinas, following his teacher Albert, sets down an outline of the various operations appropriate to the study of the heavenly bodies, in his commentary on the passage. Aquinas is familiar with the Ptolemaic system, and his remarks apply directly to it; we will therefore use them as a guide for our study of the methodological elements of this science.

> Quod autem sint plures motus huiusmodi astrorum, tribus modis deprehenditur.
>
> Est enim aliquis motus apprehensus a vulgo visu.
>
> Est et alius motus qui non deprehenditur nisi instrumentis et considerationibus. Et horum motuum quidam comprehenduntur longissimis temporibus, et quidam parvis.
>
> Est etiam tertius motus, qui declaratur ratione; quia motus stellarum errantium, invenitur quandoque velocior, quandoque tardior; et quandoque videtur esse planeta directus, quandoque retrogradus. Et quia hoc non potest esse secundum naturam córporis caelestis, cuius motus debet esse omnino regularis, necesse fuit ponere diversos motus, ex quibus haec irregularitas ad debitum ordinem reducatur.[18]

According to this analysis, there are three major modes of inquiry in the investigation.

First, and fundamentally outside any scientific research, is a mode *vulgo visu*, what is available to experience unaided by any special instruments or techniques. In the remaining two modes, we find a distinction between the work of empirical investigation, and the development and use of principles in the explanation of the results of such investigation. Aquinas distinguishes, then, a second mode *instrumentis et considerationibus*, and a third mode, *qui declaratur ratione*.

These latter two modes define the work of the professional mathematical astronomer of antiquity. In the case of the mode *instrumentis et considerationibus*, the text makes a distinction between two subordinate operations explicit. Certain observations can be made with the instruments, and recorded directly; others lead to operations of interpolation and extrapolation, giving data which cannot be directly observed, within the period of time available to the individual observer.

The mode *ratione* is the area of investigation dealing with the hypotheses; in the case of Ptolemy, as already noted, these are the epicycle and the eccentric. Once these hypotheses are formulated, and theorems concerning them worked out in a strict geometrical manner, they must be applied to the data obtained by the second mode.

This application, by which the planetary anomalies are reduced to the required order (*ad debitum ordinem reducatur*), is the major step in the Ptolemaic synthesis, and is a part of the methodology not made fully explicit in the passage in the *Metaphysics*. It is described elsewhere[19] by Aquinas as an *applicatio formalis ad materiale*, following the terminology of the *De Trinitate* passage quoted above. The application — the key to the specialized techniques of the middle science of astronomy — is effected through a transformation of both the geometrical hypotheses and the observational data into numerical expressions, which are then related.

18. *In XII Metaph.*, lect. 9, n. 2565.

19. *In I Post. Anal.*, lect. 25, n. 208.

The individual treatises of the *Almagest* follow the methodological outline suggested by Aquinas. First, the relevant phenomena are discussed in an opening section or two. Then, the hypotheses are described which will be used in the account of the planet in question. Finally, these preliminary investigations are brought together, by way of a trigonometrical calculus. The result is a statement of the size of the epicycle in relation to that of the deferent (the large circle on which the smaller epicycle is carried), the degree of eccentricity of either the eccentric, or of the deferent in the case of the combined hypothesis, and the position of apogee and perigee.

These latter conclusions are considered the key elements in a study of a particular planet. In a final section of many of the individual treatises, these results are submitted to a process of verification. Nowhere in the treatise, (save, almost accidentally, in the case of the sun) does Ptolemy present us with a suggested actual orbit of a planet with the loops representing retrogradation periods.

1. *Instrumentis et considerationibus*

A consideration of the mode of inquiry *instrumentis et considerationibus* must deal with two basic subjects. The first relates to the manner in which, by the use of instruments, the sensible characteristics of astronomical phenomena are reduced to numerical data susceptible of mathematical manipulation. The second involves the nature of this manipulation.

An awareness of the diurnal motion of the sun, or the fixed stars, requires only the simplest kind of reflection upon our experience; the annual passage of the sun, marked by the seasons, is only slightly more removed from our everyday experience. But the delimitation of the various constellations, or the different seasons, is not something amenable to "ordinary sense perception". Again, the passage of a planet through the zodiac can be followed from night to night, and the periods of the various planets can thus be determined within reasonable bounds by repeated observations and a good memory. But to accurately determine the various elements of planetary retrogradation, such as station points, or the precise period of the phenomenon, is beyond the power of unaided observation.

The gradual accumulation of observational experience led to a growing body of data, which recorded the various celestial phenomena more and more precisely. Ancient astronomers were led to develop specialized instruments to facilitate, and render more accurate, the observation and recording of these phenomena. These instruments, such as the astrolabe and the armillary sphere, were relatively complicated. Essentially, they consisted of a number of concentric metal circles attached to one another at given angles. The basic structure, composed of the fixed circles, representing the ecliptic and the meridian, for example, was coupled with other, moveable circles with sightings for viewing the planet or star. The whole apparatus, in fact, was thus set up in explicit relation to the law of circular motion which also governed the hypotheses used in the development of the theory.

The sensible elements involved in the actual observation are transformed, in this stage of inquiry designated *instrumentis*, into numerical expressions stated in terms of degrees and parts of degrees; the transformation is effected through markings on the instruments used in the observations. Through careful readings such phenomena as longitudinal variation, marking the wanderings of the planet above and below the ecliptic, were given precise values.

The second operation of this mode, *considerationibus*, can be seen through a discussion of the phenomena relative to the sun. The first figure needed, for an accurate description of the solar phenomena, is one for the length of the solar year. This is calculated from one tropic or equinoctial point to the same one in a succeeding year. A first approximation to the figure was 365 days, then 365 and a quarter days; the latter was then perceived to be somewhat too large a figure. Ptolemy therefore begins work with observations recorded by Hipparchus, 300 years earlier, and with observations made by himself; he is thus able to distribute any errors of observation over a relatively long period. There are two steps to the procedure. First, a reading of the sun's position at a critical moment — whether at a solstice or an equinox — must be made, according to the direct procedure of the mode *instrumentis*. Second, the figure obtained for the period is distributed over the 300 years of the interval in question, and the result is compared with the figure of 365¼ days which is being corrected. A difference of 12' is discovered for each year, and the figure for the solar year is thus determined to be 365 days, 14'48". This result is arrived at *considerationibus*, that is, by the use of the various techniques known to the Greeks under the general name of logistics. It is based upon the preliminary work *instrumentis*. Hence the two phases of inquiry are reasonably placed within a single mode, although they represent quite different intellectual operations. Their combined effect is to reduce all observational information to numerical form.

A further specific result of Ptolemy's observational work on the sun is a figure for each of the seasons: spring 94 days, summer 92 days, fall 88 days, and winter 90 days. These are, in fact, the critical figures for determining the orbit of the sun. Before proceeding to that determination, Ptolemy interposes an account of the relevant hypotheses. In the case of the sun, they are quite simple; their elaboration belongs to the mode *ratione*.

2. *Ratione*

In this mathematical phase of astronomical investigation, the universality of viewpoint permits the demonstration of very general properties appropriate to the epicyclic and eccentric hypotheses. The diagrams associated with them are geometrical in nature, and the demonstrations of the different properties of the hypotheses proceed in geometrical fashion. The elements specific to the theorems, represented in the diagrams, are given a physical interpretation. This constitutes a kind of delimitation of the sphere of application of the theorems, and prepares for their use in astronomical contexts. However, as we have already noted, Ptolemy gives no consideration whatsoever to the manner in which our understanding of this quasi-physical object, in its material being (whether aithereal or of some other nature) may involve a concomitant restriction of the universality of the geometrical statements. The mind, therefore, retains its grasp of the universal, mathematical argument. This grasp, however, is made possible by the avoidance of such conflicts as arise in connection with the mode of intersection of the different spheres; the failure to come to terms with these questions is an essential element in the whole tradition of "saving the appearances" discussed by Duhem.

A brief discussion of Ptolemy's presentation of the theorems to be used in later parts of the treatise on the sun will make these comments clearer. He begins his inquiry by noting that the interpretation, as we have called it, is only the first step in the astronomical work with the mathematical hypotheses. "We shall briefly show, he says, in a systematic way, first by reasoning, and second, by the numbers discovered in the appearances of the sun's anomaly, that with the above assumptions, the same appearances agree with either hypothesis." [20]

The "reasoning" is the physical interpretation of the mathematical theorem in all its universality; that is, the activity is speculative, or, as Aquinas puts it, rational, rather than logistical. The theorems demonstrated are stated in astronomical terms, although they have a geometrical analogue; they deal with certain properties of the orbit which can be studied by way of angles, and other elements of the geometrical study of the circle. For example, the first theorem stated by Ptolemy claims that "the greatest difference between the regular movement and the apparent irregular movement (difference by which the mean passage of the stars is apprehended) occurs when the apparent angular distance cuts off a quadrant from the apogee." [21] In the figure accompanying the development of the theorem, (cf. figure 1) the points marked on the circle possess physical significance — they mark the points of mean motion. The lines in the figure, on the other hand, are simply part of the constructions needed for the geometrical proof, which deals with the angles formed by them.

The theorem is demonstrated, based upon the law of uniform circular motion, and the circular hypotheses themselves. The physical phenomenon referred to in connection with it is not an observation, but instead an inference from higher principles. A single diagram, then, represents both the physical elements involved in the planet's motion, and the geometrical construction needed for the proof. The ambiguity is in part due to Ptolemy's inability to accurately determine the distances of the planets. Once this factor is added, in later times, further elements of the diagram taken on a certain physical significance, in that its representation of the spatial situation of the different heavenly bodies becomes more explicit.

It is of critical importance for the understanding of Aquinas' qualification concerning mathematical astronomy, in the *Summa* passage, to realize that Ptolemy claims that *either* of the two hypotheses, the eccentric or the epicyclic, will account for the phenomena. The hypotheses are in fact equivalent only with respect to the apparent distance covered; they in no way correspond in terms of the distance from the earth which they suggest. In the absence of a means to determine linear distance, there is no convincing criterion whereby a choice can be made between the two alternatives. It would seem, from this, that a realization of the inherent limitations of the enterprise was present in continuing astronomical research in classical times.[22]

20. PTOLEMY, *op. cit.*, III, ch. 3, p. 88.

21. *Ibid.*

22. A notable exception to this trend occurred in the work of the Arabs, in the eleventh century; cf. DREYER, *op. cit.*, ch. 9–11.

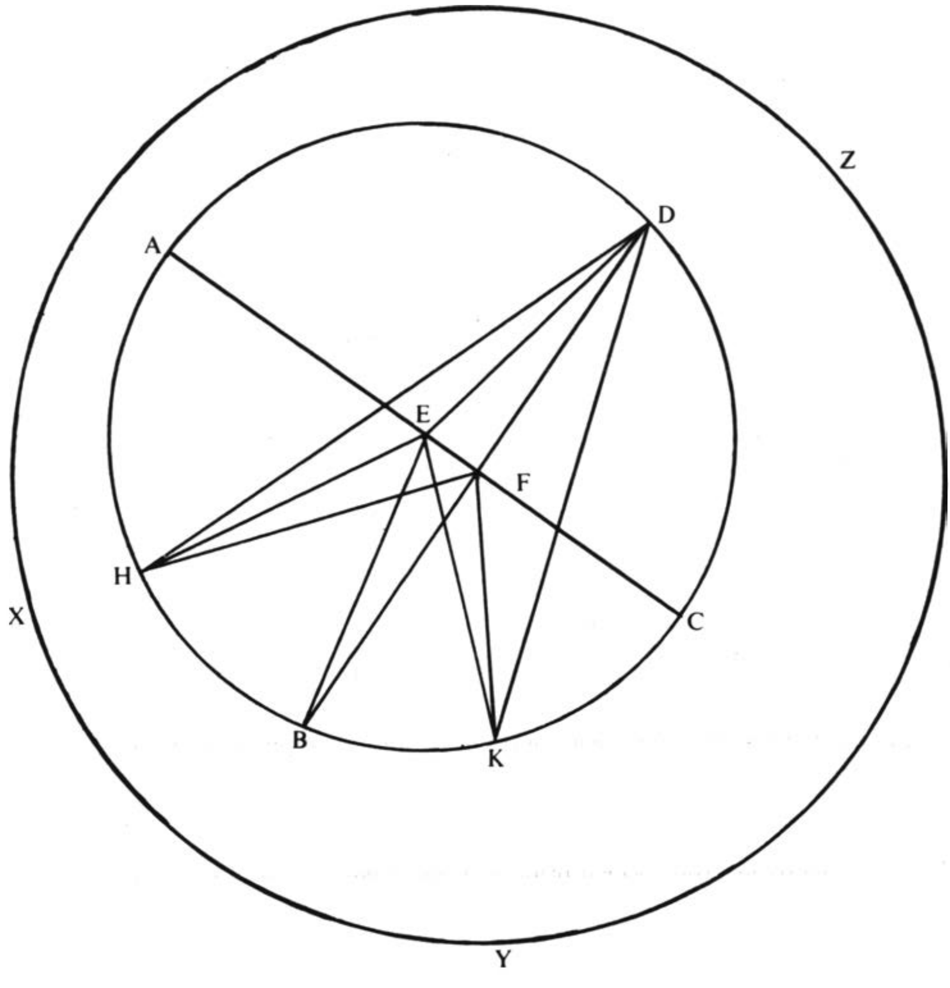

FIGURE 1 : The circle ABCD is the sun's eccentric circle. The large circle XYZ represents the ecliptic, whose center is F. B and D are points of mean motion.

The astronomer found himself confronting the fact established in logic, that a true conclusion can be deduced from both true and from false hypotheses, or premisses — although only one of these deductions constitutes a proof — and the further fact that he could not make an adequate judgement as to the truth or falsity of the hypotheses he was using. He was thus unable to determine whether he was, in fact, in possession of a proof, and could therefore only claim for his argument a value in "saving the appearances", a fact Ptolemy recognizes in another passage of the *Almagest.*[23] As Aquinas comments, *etiam forte alia positione facta [apparentia sensibilia] salvari possent.*

23. *Op. cit.*, IX, ch. 2 ; cf. also XIII, ch. 1.

3. *Applicatio formalis ad materialem*

The actual application of the mathematical hypotheses to the phenomena is distinct from the activities proper to the former modes of inquiry; it marks the essential act of the middle science of astronomy. Indeed, very little of the mathematical part of the *Almagest* appears to be the work of Ptolemy, and while he was an extremely diligent and careful observer, his work could only be of value as part of the whole corpus of observational material dating back hundreds of years. This application — "by the numbers discovered in the appearances of the sun's anomaly" — constitutes a synthesis of the two former modes. Fundamentally, Ptolemy was a brilliant synthesizer, fully in possession of his mathematical tools, and of a remarkable insight into their explanatory potential.

We have already pointed out two different uses of the mathematical diagram representing the hypothesis appropriate to the study of a given anomaly. The *applicatio formalis ad materialem* involves a third interpretation of the diagram. In it, the various parts of the figure are given precise values, derived from the observations relating to a particular planet, in order to permit the calculation of values for other parts. As in the former case, so here, the diagram is fully withdrawn from any direct relation to natural considerations. It represents an individual case of the geometrical situation lying at the foundation of Greek trigonometry.

In this third interpretation of the diagram, the trigonometric situation, appropriate to the particular case at hand, is simply reconstructed, a procedure facilitated by the preliminary trigonometric work of the first book of the *Almagest*. In the case of the first, eccentric explanation in the treatise on the sun, Ptolemy is seeking two values, which, once obtained, will permit the resolution of the sun's single anomaly. He must determine the ratio of eccentricity, and, further, he must determine the size of the angle which gives the position of the eccentric's center, and thus the line of apsides of the sun's "orbit". (Cf. figures 2 and 3). Figure 2 represents the diagram at the level of the hypothesis; figure 3 introduces considerable additional elements, needed in the trigonometrical working out of the solution.

The solutions are precise, individual values. The sun's eccentricity is approximately 2½ parts where the radius of the eccentric is 60 parts; the line of apsides GEI is inclined to the line marking the tropics, BED, by an angle of approximately 24° 30'. The fundamentals of the theory of the sun are thus established; the position of the sun's orbit has been found, in the plane of the ecliptic. The lines constructed to aid in the calculations can be neglected, leaving points marking the centers of the two circles, and the demonstration has resulted in a figure which *can* be interpreted as representing the spatial movement of the sun with respect to the earth at the center of the heavens. It is important to repeat, however, that only in the case of the sun is such a simple conclusion possible; in the case of the other planets, Ptolemy gives us the necessary figures for drawing the figure *for an instantaneous position*, never attempting to draw a figure for an entire orbit (although this too is possible).

Immediately after determining the solution of the sun's anomaly, on the eccentric hypothesis, Ptolemy takes up the same question on the epicyclic hypothesis. The second account is more complicated, in that it involves two circles. To transfer to it from the eccentric account, a simple ratio is needed: as the eccentricity is to the radius

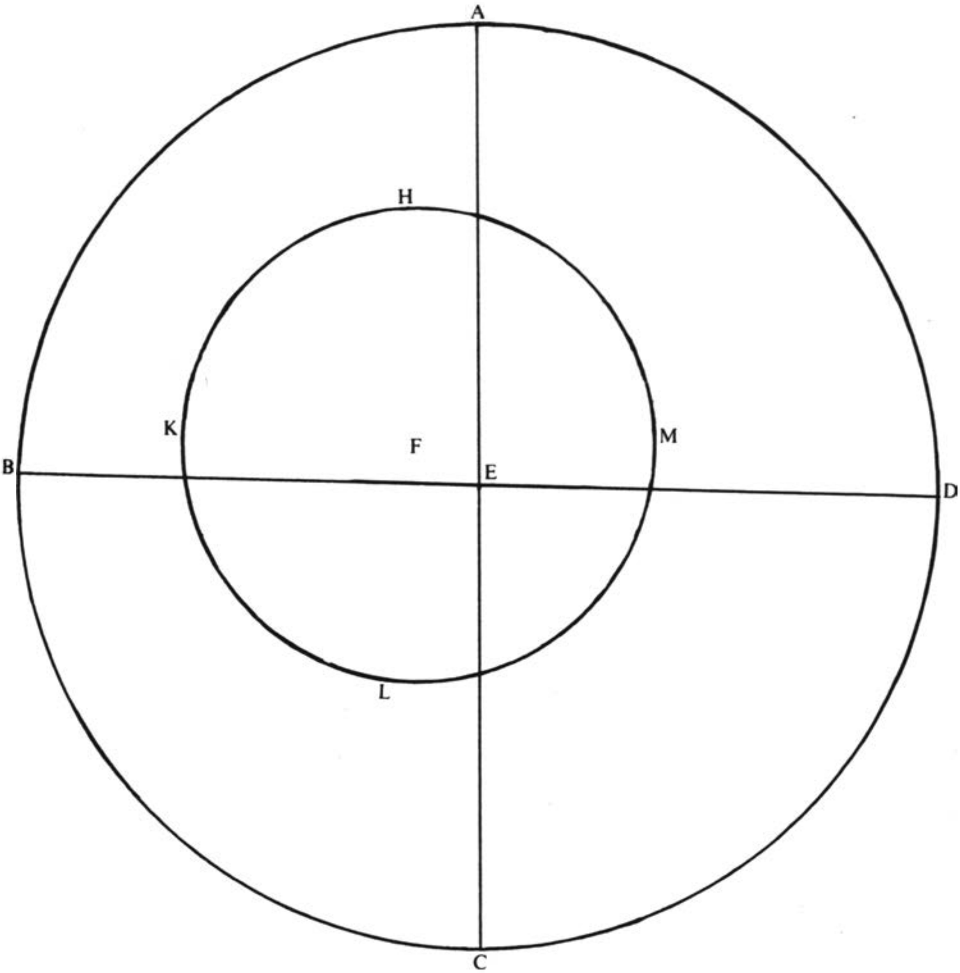

FIGURE 2 : The circles have a quasi-physical interpretation, as representing the celestial sphere (ABCD) and the eccentric circle HKLM of the sun's orbit. The lines, intersecting at right angles at E, cut off equal angles of the ecliptic circle, and form part of a geometrical interpretation having no physical counterpart (as lines).

of the eccentric, so is the radius of the epicycle to the radius of the deferent. The ratio is justified by the mathematical demonstrations of the mode *ratione*. But here Ptolemy introduces a theory which no longer has the simplicity of the explanation via the eccentric, and which has all the theoretical difficulties of his accounts of the other planets, primarily relating to the manner in which the different spheres intersect; in the eccentric account, there is only one sphere. As with the planets, again, no orbit can be — at least, no orbit is — drawn.

This presentation seems to suggest strongly that even in the simple case of the explanation of the sun's anomaly via the hypothesis of the eccentric, Ptolemy does not

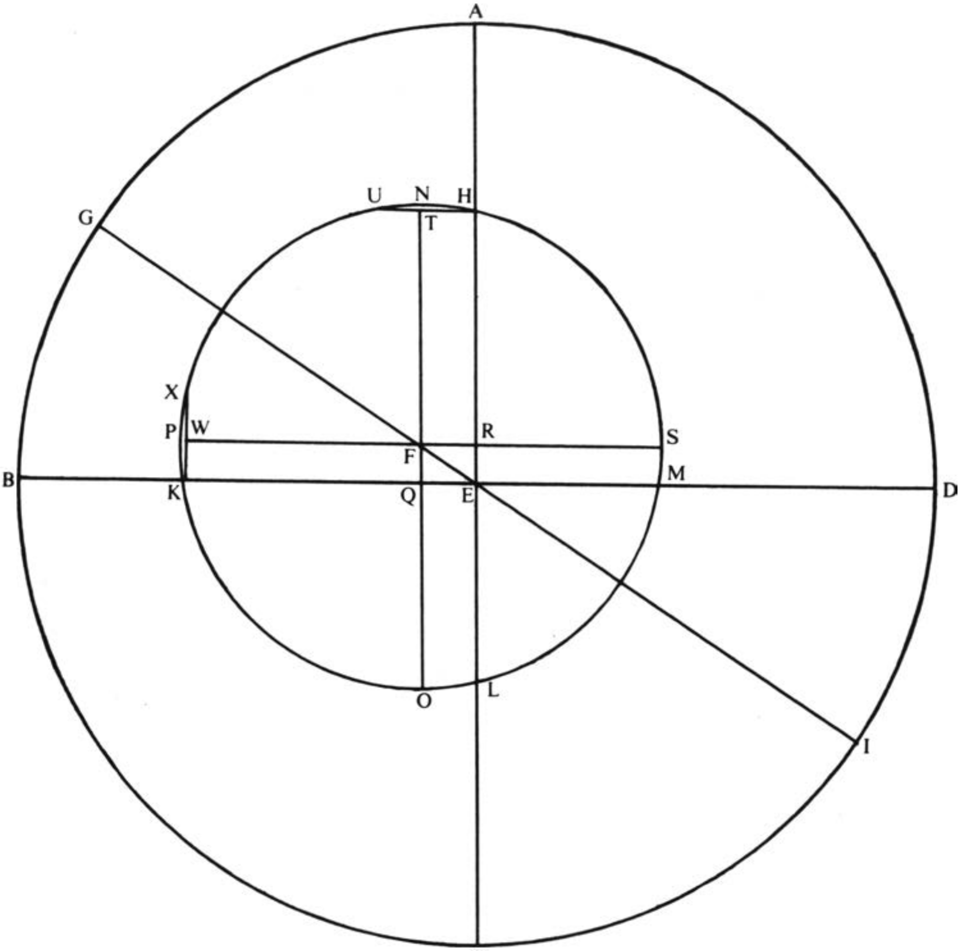

FIGURE 3 : The circles have the same interpretation as in Figure 2, as do the lines defining the quadrants. The new lines are needed for the resolution of the particular, individual problem being studied by Ptolemy. Because they are given values, and thus rendered individual, their consideration is not proper to the universal considerations of the science of geometry.

feel that he is giving an adequate (i.e. true) account of the real orbit. Too much remains unknown.

CONCLUSION

When Kepler is finally able to remove the difficulties of such an account (neglecting three-body effects, it should be added), by showing that the orbit is an ellipse with one of its foci in the sun, the movement of classical mathematical astronomy takes a new turn. The search is seen to have been for an adequate account of a particular phenomenon, and causes must then be sought for the mass of these phenomena dealing with all the planets, and with both celestial and terrestrial observations of bodies moving through space. Kepler's "laws" embody the scientific inductions from an experience much more complicated than those used in many of Aristotle's particular treatises.

The complementarity between the two traditions, which Aristotle stresses in the *Metaphysics* and the *De Caelo* as the proper relation between mathematical and natural astronomy is once again recognized, after an interval of almost two millenia. For with Kepler it is clear that the object of his science is to study the local movement of the celestial bodies, particularly the planets, and to accurately describe their orbits. In the absence of the possibility of this latter objective, ancient astronomers were forced to devise rather arbitrary hypotheses whose application could not always be defended (for example, when both applied equally to the phenomena obtainable). Such appears to have been the origin, colored with certain Platonic principles concerning the very possibility of gaining any true knowledge of the natural world and its objects, of the (quasi-mathematical) tradition of "saving the appearances". Nonetheless, the inquiry is by its very nature appropriate to the studies of the natural scientist, in the Aristotelian tradition.

Because of the extremely limited experience of celestial phenomena accessible to the observer in classical times, a natural inquiry, beyond the study undertaken in the *De Caelo*, was simply not feasible. Instead, mathematical astronomy (a specialized field of inquiry properly directed to natural concerns), undertook the enormous task of seeking to gain that experience. It slowly evolved the complex and systematic structure which we have attempted to describe. Its ultimate objective was an understanding of the real orbits of the planets, in contrast to the appearances which they present to the terrestrial observer. The hypotheses developed by the Greeks were designed to give the best possible account of these appearances. As Ptolemy was well aware, these hypotheses could be, and indeed were, changed when a change was required to more accurately "save the appearances"; it was inconceivable to consider "changing" the appearances.

But the status of the hypotheses as principles of explanation is a curious one; they are, in fact — though not necessarily in the minds of classical astronomers — attempts to gain inductive knowledge of experience which the mode *vulgo visu* could not obtain. They are thus more general than the observations they bring into focus, and give them a rationale. But they are themselves in need of explanation via natural principles.

Newton's attempt in the *Principia* is in fact to go beyond the general "descriptions" provided by Kepler and Galileo for celestial and terrestrial local motion. One major explanatory principle in his work is the principle of inertia. The principle is stated as the First Law, which reads: "Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by forces impressed upon it." With the enunciation of this law, the rejection of the fundamental principle of ancient natural philosophy of local motion, that of circularity, is complete.

It is not clear whether the work of Newton is to be looked at in a rather sceptical, positivistic manner. Much of modern physics has indeed provided researchers with profound puzzles, on the theoretical level, and there is a clear tendency to approach theoretical attempts at explanation with an attitude which ultimately has its roots in the Platonic view to which we alluded at the beginning of this essay. Nevertheless, a good many physicists of the twentieth century have concluded that a more comprehensive and "realistic" view is needed. It has been the purpose of this essay to deal with certain aspects of the history of astronomy that could serve as precedent for such a scientific realism.