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See table of contents

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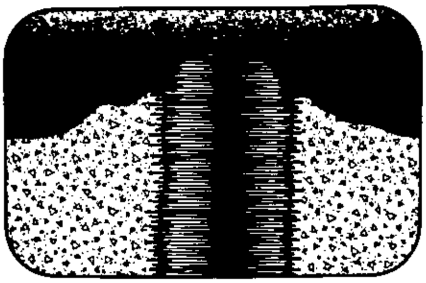
Article abstract

Simple conductive geotherm calculations provide a useful way of illustrating the thermal constraints on metamorphism in teaching senior undergraduate classes. In this article a variety of one-dimensional problems is examined to show the effects of variation in conductivity, heat production and mantle contribution, and also to show the time needed for a rock column to reequilibrate thermally after erosion.



The Thermal Background to Metamorphism - 1. Simple One-Dimensional Conductive Models

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Summary
Simple conductive geotherm calculations provide a useful way of illustrating the thermal constraints on metamorphism in teaching senior undergraduate classes. In this article a variety of one-dimensional problems is examined to show the effects of variation in conductivity, heat production and mantle contribution, and also to show the time needed for a rock column to reequilibrate thermally after erosion.

Introduction
In advanced undergraduate classes in metamorphic petrology the instructor is faced with the problem of explaining the thermal link between tectonics and metamorphism. This topic is often avoided as being too complex or too mathematical, yet an understanding of the thermal background to metamorphism is essential to a proper appreciation of how and why metamorphism occurs. The purpose of this article is to show how a simple set of one dimensional thermal models can provide a useful understanding of the basic controls on metamorphism. Some of these models are mathematically accessible to the average student and can be set as assignments. The second article in the series will deal with more complex and realistic two dimensional models. These are mathematically too difficult for most students to compute, but a study of the results they give should lead to an intuitive feel of how rocks behave thermally.

The work reviewed in these articles is based on that of various authors, especially those formerly working in Oxford, including amongst others Bickle (1973), Bickle *et al.* (1975), Oxburgh and Turcotte (1974), and the important later work of England and Richardson (1977). These authors have applied the results of Carslaw and Jaeger (1959) to a variety of metamorphic problems. Thompson (1981) has recently summarized much of this work.

The thermal structure of any rock pile can be influenced by a wide variety of factors, both "internal" - i.e., properties of the rock pile, and "external" - controlled from outside the pile. To investigate the relative importance of these effects we shall first review the controls on the geotherm existing in a simple vertical pile of rocks; then we shall consider the way in which a metamorphic rock is brought to the surface. We do not attempt an exhaustive survey of the many possible ways in which metamorphism can occur (Turner (1968) stressed the unique character of each individual metamorphic belt); rather we hope to be able to show how the various parameters controlling rock temperatures can act together in a variety of ways to produce metamorphic rocks.

The Significance of the Geotherm
The geothermal gradient in a rock column is controlled by several parameters, some internal to the rock column and some external. Internal parameters include the conductivity and heat capacity of the rock, as well as the radioactive heat generation, while external factors include the heat flow into the column from below and the erosion rate at which material is removed from the top of the column. If a column is not being eroded or deposited upon, and if basal heat remains constant, the column may eventually reach a state of thermal equilibrium in which the temperature at any given point in the column is steady. If the column is not in an equilibrium state, the initial temperature gradient when the column was constructed is important, as is the time interval since the construction of the column. To demonstrate the importance of these various factors we have calculated their effect on the "geotherm" of a typical column of rock. It is a useful class exercise to ask a student to calculate a geotherm, in order to understand the significance of the various

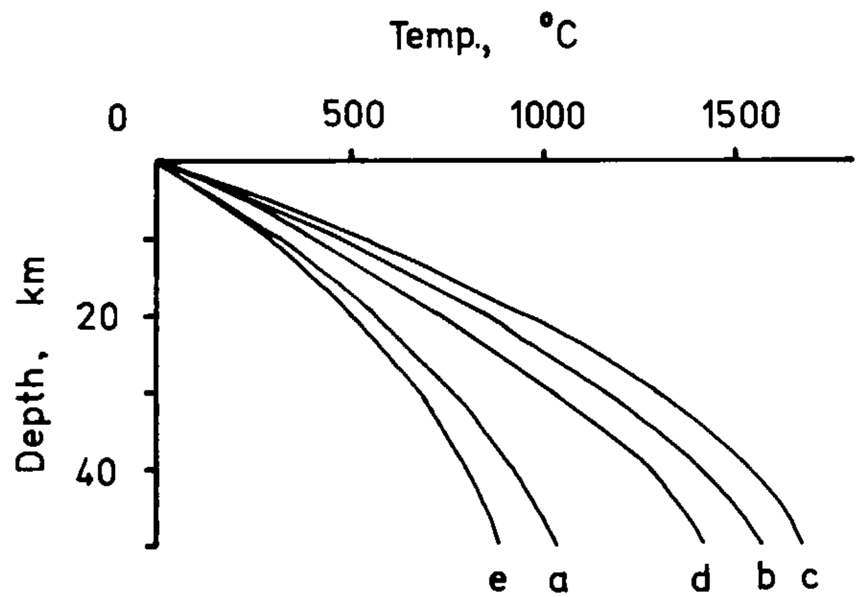

Figure 1 *Equilibrium (a-e) and erosional (f, g) geotherms for a 50 km thick column of rock. Standard model has conductivity k = 0.006 cal cm^{-1}sec^{-1}°C^{-1};* (2.52Wm^{-1} K^{-1}) internal radioactive heating, A = *3.0 h.g.u. (1 h.g.u. = 0.42 x 10^{-3} milli Wm^{-3} = 10^{-13} cal cm^{-1}sec^{-1}); and deep heat contribution from mantle and lower lithosphere q_m = 0.5 h.f.u. (1 h.f.u. = 41.8 milli Wm^{-2} = 10^{-6} cal sec^{-1}cm^{-2}). a) Standard model. b) Standard model, but with k = 0.004 cal cm^{-1}sec^{-1}°C^{-1}* (1.68Wm^{-1} K^{-1}). c) Standard model, but with A = *6 h.g.u. (2.51x10^{-3} mWm^{-3}). d) Standard model, but with q_m = 1.0 h.f.u. (41.8mWm^{-2}). e) Standard model, but with q_m = 0.3 h.f.u. (12.5mWm^{-2}). f) Standard model, eroded at 1 km/Ma for 25 Ma, then with no further erosion. 0 - represents standard geotherm of Figure 1a; "25" is geotherm immediately after erosion for 25 Ma; "100" is after 100 Ma; unlabelled dashed line is final equilibrium geotherm. Dotted horizontal line is new surface model. g) Standard model deposited upon by sediment at a rate of 0.5 km/Ma for 25 Ma. Notation as in (f). h) Two layer model 'Archaean' geotherm. Crust 35 km thick, top 20 km has heat production of 10 h.g.u. (4.18x10^{-3} mWm^{-3}), lower 15 km has heat production of 2 h.g.u. (.84x10^{-3} mWm^{-3}). Heat flow from mantle = 1.5 h.f.u. (62.7mWm^{-2}).*
(Figure 1 continues on the next page.)

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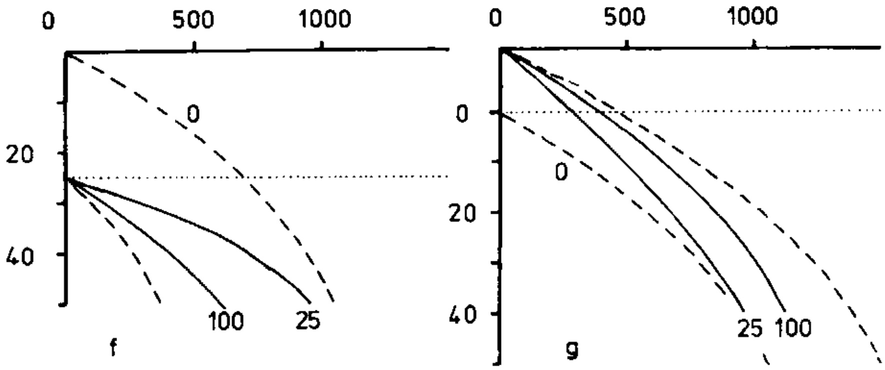

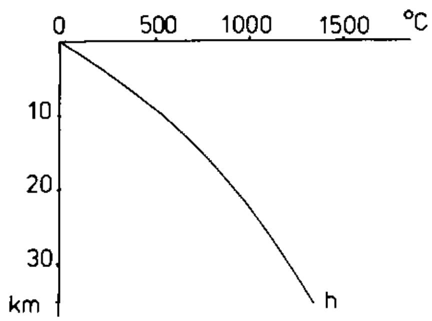

Table I. *Heat Production of Average Rock Types*

Rock Type	Mafic Igneous	Diorite, Quartz Diorite	Granodiorite	Silicic Igneous
U contribution (cal $gm^{-1}yr^{-1}$)	$.7 \times 10^{-6}$	1.5×10^{-6}	1.9×10^{-6}	3.4×10^{-6}
Th contribution	$.5 \times 10^{-6}$	1.7×10^{-6}	1.9×10^{-6}	4.0×10^{-6}
K contribution	$.1 \times 10^{-6}$	$.3 \times 10^{-6}$	$.7 \times 10^{-6}$	1.1×10^{-6}
Density (gm cc^{-1})	2.9	2.8	2.7	2.7
Heat Production (h.g.u.)	1.2	3.1	3.8	7.3
($mWm^{-3}.10^{-3}$)	.5	1.3	1.5	3.1

Mean values from Clark, Peterman and Heier, and Wetherill (in Clark, 1966). One h.g.u. = 10^{-13}cal $cm^{-3}s^{-1}$ = 0.418×10^{-3} mWm^{-3}.

parameters. Appendix I lists a suitable simple equilibrium geotherm problem.

A) Equilibrium Geotherm

We take a "typical" rock column 50 km thick, of conductivity 0.006 cals cm^{-1} $°C^{-1}sec^{-1}$ (2.52Wm^{-1} K^{-1}), heat capacity that of the empirical formula of England (1977), internal radioactive heating 3.0 h.g.u. (1.26x10^{-3} mWm^{-3}), with a heat flow contribution from depth of 0.5 h.f.u. (20.9mWm^{-2}) at the base of the column and undergoing no erosion or deposition. The equilibrium temperature structure of this column is shown in Figure 1a; at shallow depth (circa 10 km) the gradient is approximately 30° C/km, while at deep level (below 30 km) the gradient is around 15° C/km or less. We now individually vary the values of the various parameters for this "typical" column to show the effect they have on the geotherm.

Conductivity. Reducing the conductivity of the whole pile to 0.004 cals cm^{-1} $°C^{-1}sec^{-1}$ (1.68Wm^{-1} K^{-1}) has the effect of changing the shallow-level equilibrium geotherm to about 45° C/km (Figure 1b). Raising the conductivity to 0.008 cals cm^{-1} $°C^{-1}sec^{-1}$ (3.36Wm^{-1} K^{-1}) would have the opposite effect, reducing the gradient to about 23° C/km at shallow-level.

Heat Generation. Increasing the heat generation to 6 h.g.u. (2.51x10^{-3} mWm^{-3}) raises the shallow-level equilibrium to over 50° C/km (Figure 1c); in contrast reducing heat generation to 1 h.g.u. (0.42x10^{-3} mWm^{-3}) reduces this (shallow-level) geotherm to about 16° C/km. Table I lists typical heat generations of average rock types.

B) The Approach to Equilibrium

The examples given above are equilibrium geotherms. In most rocks the conductivity is so low that equilibrium takes a very long time to attain: it is very important in undergraduate teaching to show that equilibrium is an unlikely state of affairs in a young mountain belt. The following examples illustrate this.

Heat Contributions from the Deep Mantle. If the heat contribution q_m from below the column of rock is increased from 0.5 to 1.0 h.f.u. (20.9 to 41.8mWm^{-2}), it takes a very long time indeed for the effect to be seen at shallow depth. The equilibrium geothermal gradient will increase (Figure 1d) towards an eventual circa 40° C/km at shallow-level (circa 10 km), but after 100 Ma the temperature is still 4% away from the new equilibrium. A rock 20 km deep would initially be at a temperature of 567° C, the equilibrium temperature with a heat flow, q_m = 0.5 h.f.u. (20.9mWm^{-2}) into the base of the column. Twenty million years after q_m changed to 1.0 h.f.u. (41.8mWm^{-2}) this rock would have only heated to about 580° C; only after 100 Ma would the temperature be over 700° C and close to the new equilibrium. This very clearly demonstrates how *slow* is the thermal response of a rock column; it would take a very long time indeed for the heat from a subduction zone at, say 200 to 300 km depth, to have a significant effect on the temperatures at a depth of 20 km if all heat transfer is by conduction alone. The concept that metamorphism is caused by "the thermal energy surging up from the depth" (Wenk, 1962; translated by Winkler, 1974) is thus unlikely, or at least a very long-term process indeed. Furthermore, large increases in q_m would cause large-scale melting at depth long before the heat had penetrated to a high level; thus a metamorphic belt caused by a deep-seated heat source would be characterised by abundant intrusions, probably of mantle-derived material, which would be the dominant factor in heat transfer to the surface (a feature not seen in the Alps, for example). Figure 1e shows the effect of reducing q_m to 0.3 h.f.u. (12.5mWm^{-2}); the reequilibrium to this new geotherm is again a very long-term process.

The Initial Temperature Gradient. When a rock column is assembled, by some process such as sedimentation, overthrusting or intrusion, the initial temperature gradient is likely to be very different from the equilibrium gradient. It is therefore interesting to see how long it will take for the equilibrium gradient to be reached from different initial conditions. If the "typical" rock column had an original gradient of, say, 25° C/km throughout, the gradient would have adjusted to within 5% of equilibrium at a depth of 20 km within 4 Ma, and to within 2% of equilibrium after 10 Ma. An initial gradient of 15° C/km would be within 10% of equilibrium at 20 km within 60 Ma, and an initial gradient of 10° C/km would take about 80 Ma to reach a temperature within 10% of equilibrium at 20 km. Thus it can be seen that the results of "anomalous" thermal gradients can last a very long time - we shall return to this point later.

Erosion and Deposition. Figures 1f and 1g show the effects of erosion at 1 km/Ma for 25 Ma on the pile, and of deposition of 0.5 km/Ma for the same period; in both cases curves are also drawn after 100 Ma to show the effect of 75 Ma of thermal relaxation. In the erosional case the shallow-level (10 km) geotherm is raised to 50° C/km after 25 Ma, after which it slowly relaxes toward equilibrium; in the depositional case the shallow-level geotherm is depressed to 23° C/km after 25 Ma, and relaxes close to equilibrium by 100 Ma. Two points emerge: first, it is clear from the above analysis that *rapid* changes to the geotherm are most easily caused by process such as erosion and overthrusting; changes in the deep (mantle) heat flow can only be important over a very long time scale. Secondly, the nature of the rock in the pile is a very significant control; granites for instance, will have a very much higher thermal gradient than mafic rocks, which contain far less of the heat producing elements.

Two Layer Model. The models presented so far have been very simple: they have assumed that the top 50 km of the earth is of uniform composition. This of course is not in any way true, but was a useful starting point. A slightly more realistic model would assume a layered continental crust, with heat production concentrated toward the top. The calculation of the geotherm in the layered model is exactly as in the simple model, except that each layer must separately be considered. Figure 1h shows an equilibrium geotherm calculated for a model 'Archaean' crust (many of Canada's metamorphic rocks are Archaean or Proterozoic in age). The model has two layers. The upper layer is 20 km thick, with an internal heat production of 10 h.g.u. (=4.18×10^{-3} mWm^{-3}). This heat production in the Archaean would decay to about 3-5 h.g.u. (1-2.5mWm^{-3}) today, depending on the relative amounts of U, Th and K in the rock, and the age of the rock. The lower layer has an internal heat production of 2 h.g.u. (.84×10^{-3} mWm^{-3}) and is 15 km thick. The basal heat flow from the upper mantle into the crust is set at 1.5 h.f.u. (62.7mWm^{-2} sec^{-1}) and the surface temperature at 0° C.

Figure 1h demonstrates clearly that Archaean geotherms should have been relatively high. Over the history of the Earth there seems to have been a progressive relative enrichment of the upper part of the crust in the heat producing elements (which tend to be relatively mobile), and the deep continental crust is probably now poor in heat production.

Significance in Undergraduate Teaching. The examples given above show the relative importance of the various controls on the thermal structure of a rock pile. While an analysis of the time-dependence of metamorphism is probably beyond the mathematical ability of the average student, it is very easy to calculate a simple equilibrium geotherm (Appendix I) and from the examples given to gain a qualitative "feel" for the length of time involved in conductive reequilibrium of a rock pile.

In the second of these articles we shall examine the more complex two-dimensional models, which provide a good illustration of the controls on regional metamorphism and the development of a metamorphic facies series.

The problem which follows is suitable for a student assignment.

Acknowledgements
Thanks to R.St.J. Lambert and J. Martignole for comments, and to P.C. England and A.B. Thompson.

Appendix I

Problem: Calculating an Aphroditotherm

A) Florensky *et al.* (1977, Geol. Soc. Amer. Bull., v. 88, p. 1537-45) report results of missions to Venus. The surface temperature was measured as 740° K and at three sites heat producing elements were measured as

	Venera 8	*Venera 9*	*Venera 10*
K	0.47±0.08%	0.30±0.16%	4±1.2%
U	0 60±0.16 ppm	0.46±0.26 ppm	2.2±0.2 ppm
Th	3.65±0.42 ppm	0.70±0.34 ppm	6.5±0.2 ppm

The density of the Venusian crust can be taken from a measurement by Venera 9 of 2.8 tonne m^{-3} ($gm\ cm^{-3}$). Using the following heat productions for terrestial K, Th, U:

$K = 2.7 \times 10^{-5}$ cal $gm^{-1}yr^{-1}$
$U = 0.73$ cal $gm^{-1}yr^{-1}$
$Th = 0.2$ cal $gm^{-1}yr^{-1}$
where 1 yr = 3.15×10^{7} secs,
calculate heat production in cal $cm^{-3}sec^{-1}$ at each site.

B) The heat equation (T = temperature, x = depth)
where A = heat production and
κ = conductivity

$$\frac{\delta^2 T}{\delta x^2} = \frac{-A}{\kappa}$$

gives $$\frac{\delta T}{\delta x} = \frac{-Ax}{\kappa} + c$$

Conductivity may be assumed to be 0.006 cal $cm^{-1}sec^{-1}$ $°C^{-1}$ (a typical value for silicates) and A is calculated above. Assume that at a depth of 50 km heat flow from the mantle and deep lithosphere of Venus is 0.5 h.f.u. (1 h.f.u. = 10^{-6} cal $cm^{-2}sec^{-1}$) and hence by using heat flow = thermal gradient x conductivity
calculate $\frac{\delta T}{\delta x}$ at x = 50 km
and hence find c.

Further integration of the heat equation gives
$$T = \frac{-Ax^2}{2\kappa} + cx + d.$$

At the surface x = 0, T = 740° K. Find d. From these values of A, c and d and the assumed value of κ, plot Venus geotherms to 50 km at each site. What are the assumptions in this calculation? What are the implications for Venusian metamorphic petrology?

Solution

A) Heat Production

Venera 8

i) κ: One cm^3 of sample contains

$$\frac{(0.47 \pm 0.08) \times 2.8 \text{ gm of } \kappa}{100}$$

Heat Production from κ is

$$\frac{0.47 \times 2.8 \times 2.7 \times 10^{-5}}{3.15 \times 10^{7} \times 100} \text{ cal cm}^{-3}\text{sec}^{-1}$$

$= 0.113 \pm 0.019 \times 10^{-13}$ cal $cm^{-3}sec^{-1}$

ii) U: Heat Production from U is

$$\frac{0.6 \times 2.8 \times 0.73}{10^{6} \times 3.15 \times 10^{7}} \text{ cal cm}^{-3}\text{sec}^{-1}$$

$= 0.389 \pm 0.104 \times 10^{-13}$ cal $cm^{-3}sec^{-1}$

iii) Th: Heat production from Th is

$0.649 \pm 0.075 \times 10^{-13}$ cal $cm^{-3}sec^{-1}$

Note: at this stage, once it has been demonstrated that they are large, the errors are ignored!

Total Heat Production =
1.15×10^{-13} cal $cm^{-3}sec^{-1}$

Venera 9

Total Heat Production =
0.49×10^{-13} cal $cm^{-3}sec^{-1}$

Venera 10

Total Heat Production =
3.54×10^{-13} cal $cm^{-3}sec^{-1}$

B) Calculation to find c and d

Venera 8

0.5×10^{-6} = thermal gradient at 50 km x 0.006

thermal gradient at 50 km = 0.83×10^{-4} °C/cm (=8.3° C/km)

Thus at 50 km, $0.83 \times 10^{-4} = \frac{-1.15 \times 10^{-13} \times 50 \times 10^{5}}{0.006} + c$

$c = 1.79 \times 10^{-4}$ °C cm^{-1}

$d = 740°K = 467°C$

Venera 9

$c = 1.24 \times 10^{-4}$ °C cm^{-1}

$d = 467°C$

Venera 10

$c = 3.78 \times 10^{-4}$ °C cm^{-1}

$d = 467°C$

Plots of $T = \frac{-Ax^2}{2\kappa} + cx + d$

are shown in Fig. 2.

8, 9, 10 refer to geotherms for Venera 8, 9, 10.

The assumptions are obvious: a) that Venus is Earth-like in its isotopic composition (probably a safe assumption) and in heat flow from the lower lithosphere (very unsafe); b) that the heat production is isotropically distributed in the Venusian crust and upper lithosphere (not true on Earth); and c) that an equilibrium geotherm is valid.

On Figure 2 is also plotted the Al_2SiO_5 triple point. Leaving aside the obvious point that metapelites are wildly unlikely on a dry planet, it is clear that high pressure/low temperature assemblages do not exist on Venus. The implications for partial melting in a dry setting are also worth pointing out.

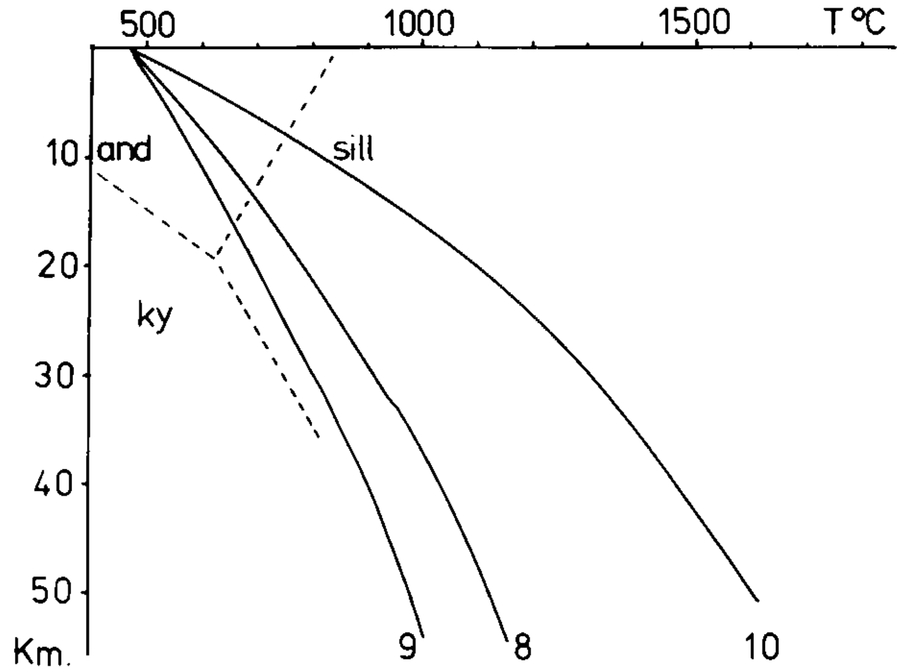

Figure 2 *Solutions to problem, showing thermal gradients deduced for Venera 8, 9 and 10, and Al_2SiO_5 triple point.*

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