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Anuj Misra

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See table of contents

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Sourcebook in the Mathematics of Medieval Europe and North Africa edited by Victor Katz

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Reviewed by
Anuj Misra*
University of Copenhagen
anuj.misra@hum.ku.dk

1. Introduction

The story of mathematics, like any good story, is about the actors and their activities. While a narrative can help weave fragmentary episodes into a coherent tale, the richness in its telling lies in the performative nuances of the characters themselves. The ways in which individual actors confront the situations that they encounter shape the stories of their interconnected histories. In the chronicle of mathematics, the probative force of this agency is most excellently demonstrated by looking at the writings of the practitioners themselves, and this is precisely what a sourcebook is meant to provide.

The *Sourcebook in the Mathematics of Medieval Europe and North Africa* brings together a selection of mathematical writings in Latin, Hebrew, and Arabic from medieval Europe and North Africa (the Maghreb). At the very outset, it offers its readers an opportunity to survey briefly the mathematical contributions of scholars writing in different languages, at different times, and in different places across the intellectual geography of the medieval West. The three main chapters of this sourcebook present a collection of topics studied by mathematicians writing in Latin, Hebrew, and Arabic from the ninth to the 15th centuries. The contributors to this sourcebook have provided, often for the very first time, English translations of excerpted Latin, Hebrew, and Arabic texts for the benefit of readers unacquainted with the source languages. These translated passages not only give readers direct

* Anuj Misra is a historian of mathematics who works on late medieval and early modern sources in Sanskrit, Arabic, and Persian. He is currently a Marie Skłodowska-Curie Fellow at the University of Copenhagen and leads the project Early Modern Exchanges in Sanskrit Astral Sciences, which examines the cross-cultural transmission of mathematical astronomy between the Persianate and Sanskritic traditions of 17th-century Mughal India.

access to the mathematical thinking of the authors, they also reveal the reticulated nature of developments in premodern mathematical thought. In fact, right in the general introduction, readers are made aware of the dilemma in situating individual mathematical works within the three chapters. The ensuing editorial choice to use the linguistic identity of the original author when known, and if not, to follow the language in which the extant material source is written, already betrays the complexity and dynamism of the exchange of knowledge between these cultures. By citing substantial excerpts from different textual traditions, this sourcebook illustrates beautifully the vibrancy of the intellectual commerce that thrived as Christian, Jewish, and Islamic scholars came to cohabit the space of medieval Western Europe.

It is perhaps also worthwhile to remember that a sourcebook is not didactic in the ordinary sense of a textbook. The paucity of introductory essays is intentional as it allows more room for reflective examination of the carefully selected source material. The editor's hope that readers "come to appreciate the mathematical struggles of our medieval ancestors and the answers they found to the problems they posed" [2] serves as a reminder that any threads connecting different episodes in the story of a subject are to be found while journeying through the episodes themselves.

Some readers may consider this to be a shortfall of the sourcebook, as it lacks a prefatory discourse connecting the selected passages into a coherent whole. Indeed, in grouping these excerpts into different subject-categories (more on this below), it is not always clear as to why these particular authors (or their works) are selected. The ontological consequence of this categorization notwithstanding, it does, however, offer some insights into the milieus in which authors posed the questions that they tried to answer.

The following remarks on the *Sourcebook*'s three chapters highlight some of the more notable aspects in their presentation of Latin, Hebrew, and Islamic mathematics. By no means do they exhaust the contents of this book.

2. The Latin mathematics of medieval Europe

The first chapter of this sourcebook surveys the contributions made by Latin (Christian) scholars of Europe from 800 to 1480. It categorizes the developments in the history of Latin mathematics into three periods—

- Latin schools, 800–1140
- A school becomes a university, 1140–1480
- Abbacist schools, 1300–1480

—with each period treated as an individual section. The collection of excerpts in each section is further categorized under a topical theme: for instance, the second section on universities includes passages on the topics of arithmetic, algebra, geometry, trigonometry, and so on, which are grouped under separate topical headings. Although not explicitly stated, a quick look at the names of the works grouped in each section will convince readers that these topical categories are not necessarily those identified by the original authors (i.e., actor's categories). For instance, excerpts from Fibonacci's *Book on Calculation* appear under both arithmetic and algebra in the section on universities. This topical (or subject-based) categorization of works in different sections is an editorial choice, in much the same way as the chronological division into three sections itself is an expository exercise.

2.1 *Latin schools* The system of representing numbers by the positions of fingers ("finger-reckoning") in the *De temporum ratione* (The Reckoning of Time) by Bede the Venerable (*ca* 672–735) provides an entertaining beginning to this section. Subsequent excerpts from theoretical texts such as Boethius' *De institutione arithmetica* (*Introduction to Arithmetic*, *ca* 500), and practical texts written for beginners in the use of the abacus such as Pandulf of Capua's *Liber de calculatione* (*Book of Calculations*, *ca* late 11th century), reveal how Latin scholarship in arithmetic was motivated by both theory and praxis. The passage from Franco of Liège's *De quadratura circuli* (*Squaring the Circle*, *ca* 1050) is particularly interesting. Franco accepts that the areas of a circle and a square are unequal in number, yet he attempts to demonstrate their equality by geometry. His attempt, although only partially successful, nevertheless shows how mathematical thinking in Latin Christian Europe began to embrace novelty in the late 11th century. Towards the end of this first section, examples of recreational mathematics in the form of number puzzles, e.g., *De arithmeticis propositionibus* (*Arithmetical Representations*) by Bede, and thought problems, e.g., *Problems to Sharpen Young Minds* by Alcuin of York (735–804), are presented. Among these, the description of the rules of the 11th-century mathematical game *Rithmimachia* or *Rithmomachia* (*Fight with Ratios of Numbers*) is as delightful as it is educational to read. It shows the pedagogical value of arithmetical games in the Middle Ages [59–64].

2.2 *A school becomes a university* In the first topic of this section, readers are presented with literal English translations of the first theorem of the first book of Euclid's *Elements* that are derived from its Greek version vis-à-vis the ones that are derived from its Latin versions (as well as from its subsequently reworked versions). The conceptual differences that arise due to different channels of transmission—for example, from Greek to Arabic to Latin, or from Greek to Arabic to Castilian to Latin, as well as those that follow from later re-workings in the 12th and 13th centuries—are made demonstrably clear to the reader [71–75].

The passages from al-Khwārizmī's *Arithmetic* (in Latin, *ca* 12th century) show how the Hindu-Arabic place-value system and the concept of zero were initially difficult to understand in the Latin West. The excerpts from John of Sacrobosco's *Algorismus vulgaris* (*ca* 1230), a popular work derived from al-Khwārizmī's *Arithmetic*, reveal the increasing significance of the study of the "art of numbering" among university students. On a more practical level, the examples from chapter 12 of Leonardo of Pisa's (*alias* Fibonacci) *Liber abbaci* (*Book on Calculations*, 1202) explain the use of arithmetic in everyday problem-solving. The problem "On the Finding of a Purse", where Fibonacci understands negative numbers as amounts to be in "debit", is certainly noteworthy. The contributors to this chapter also point to a commonality in the nature of this problem as it first appears in India (Mahāvīra's *Gaṇitasārasaṃgraha*, *ca* eighth century [563–64: appendix 3]) and subsequently manifests in numerous Islamic and medieval Latin works.

Turning to algebra, the small excerpts from the translation of al-Khwārizmī's *Algebra* (*ca* 830) by Robert of Chester (*ca* 1145) show how concepts, methods, and proofs came to define the study of Latin Algebra in the High Middle Ages. In the selections from Fibonacci's *Liber abbaci*, we see Fibonacci's "direct method" using algebra (as opposed to arithmetic) in solving practical problems. Some readers may find particularly interesting the geometrical justifications that Fibonacci uses in describing his "double false position" method of solving equations of the form $mx + r = c$ (from chapter 13), as well as those that he gives for his method of solving "squares plus numbers equal roots" (from chapter 15) [107–112].

There are several captivating episodes that highlight developments in the topics of geometry and trigonometry in Latin Europe. For example: the proof of Heron's formula in Gerard of Cremona's translation of the *The Book of the Measurement of Plane and Spherical Figures* (*ca* late 12th century) by Banū Mūsā; Fibonacci's algebraic explanations on measuring fields of varying shapes in his *De practica geometrie* (*Practical Geometry*, 1220); or even Ptolemy's derivation of chord lengths in a circle of radius 60—including his own proof for "Ptolemy's Theorem"—from his *Almagest* (*ca* second century). The discussions on planar and spherical trigonometry from Regiomantanus' (*alias* Johannes Müller) *De triangulis omnimodis libri quinque* (*Five Books on Triangles of Every Kind*, *ca* 1462/64) deserves particular mention.

On the notion of infinity in Latin Europe, the excerpts from Campanus of Novara's reworking of Euclid's *Elements* (*ca ante* 1259) and Thomas Bradwardine's *Geometria speculativa* (*ca* early 14th century) show how infinitesimals were interpreted geometrically: that is, how a non-rectilinear or horn-like angle—called an *angulus contingentiae* (angle of contingence)—between a tangent and the circumference of a circle at the point of tangency could be infinitely divided into smaller parts. In fact, in his *Tractus de continuo* (*On the Continuum*, *ca* early 14th century), Thomas Bradwardine offers an excellent review of the scholarly opinions on continuity and discreteness prevalent at his time.

With his cautionary passage, Robert Grosseteste spelled out the separation that he felt needed to be maintained between mathematics and the experiential world in his *De lineis, angulis et figuris* (*On Lines, Angles, and Figures*; *ca* early 13th century). For Robert, the corporeality of the world was evident in movable matter and, hence, explicable by the science of mechanics; geometry was just the language of mechanics. Building on this idea, scholars at Merton College in Oxford wrote works on kinematics (spatial and temporal changes in movement: the effect) and dynamics (forces producing the changes: the cause) that came to influence mathematical physics of late medieval Europe. Excerpts from the chapters in part 1 of Nicole Oresme's *De configurationibus qualitatum et motuum* (*On the Configurations of Qualities and Motions*, *ca* 1350) are remarkable as they show how Oresme used geometrical graph-like figures with rectangular coordinates to represent changing (uniform and non-uniform) distribution of various quantities before Descartes.

2.3 *Abbacist schools* The third and last section of this chapter is on the Abbacist schools of northern Italy (1300–1480) that helped educate members of the mercantile class. With Florence becoming the center of European banking, the practical curriculum in mathematics taught at Florentine Abbacist schools—in vernacular Italian instead of Latin—was designed for commercial applicability. For instance, excerpts concerning the calculation of foreign exchange illustrate the need for merchants to understand the Rule of Three; and excerpts about land measurements demonstrate the need for practical geometry. In fact, the passages from the lecture notes of Gilio da Siena (*flor*. 1374–1407) show what students might have learned in a course on introductory algebra at an Abbacist school.

Overall, the chapter on Latin mathematics includes many interesting excerpts that readers might find enjoyable to discover. A few such examples are cited here: the allegorical way of interpreting numbers as mysterious (or sacred) in Isidore of Seville's *Liber numerorum* (*Book of Numbers*); the combinatorial calculations in the anonymous poem *De vetula* from *ca* mid-13th century France; the "shadow square" (a numbered U-shaped figure on the back of an astrolabe) in the English poet Geoffrey Chaucer's *A Treatise on the Astrolabe* from the 14th century; the mathematical impossibility of "angels" being impelled by continuous motion in a lecture on indivisibles and theology by John Duns Scotus (*ca.* 1266–1307); the first use of geometrical diagrams representing a "function" in Giovanni di Casali's *De velocitate motus alterationis* (*On the Velocity of Motion of Alteration*, from 1346); and the unique solution of cubic and quartic equations proposed by Master Dardi (*flor*. 1344).

3. Mathematics in Hebrew in medieval Europe

The second chapter of this sourcebook discusses the contributions made by Hebrew (Jewish) scholars of mathematics between the 11th and 16th centuries in Europe. Beginning with a chronology of Hebrew mathematical texts written in this period, the chapter proposes to analyze the historical development and continuity in these works by categorizing them into five thematic sections with increasing complexity:

- Practical and Scholarly Arithmetic
- Numerology, Combinatorics, and Number Theory
- Measurement and Practical Geometry
- Scholarly Geometry;
- Algebra.

The contributors to this chapter state that the English translations of (most) Hebrew excerpts are directly based on primary sources (manuscripts or editions). Readers unfamiliar with the history of Hebrew mathematics (or the language of Hebrew) should find this information reassuring. The chapter's contributors are scholars of repute and hence their translations adhere to rigorous academic standards.

3.1 *Practical and scholarly arithmetic* Excerpts from the foundational text of Abraham ibn Ezra, *Sefer Hamispar* (*The Book of Numbers*, *ca* 12th century) describes the arithmetic of numbers and fractions, geometric and harmonic ratios, calculations of square roots, calendrical and commercial problems using proportions, and techniques of geometrical mensuration, among other topics. The translations show how Ibn Ezra's instructions on some of these topics, including his explanation of the Hindu-Arabic decimal system, are interwoven with Jewish mysticism.[1] Immanuel Ben Jacob Bonfils' description of sexagesimal division that contextualizes the treatment of decimal fractions, or Rabbi Jacob Canpanṭon's iterative algorithm to extract the roots of integers (*ca* 14th–15th century), illustrate aptly how medieval Jewish scholars combined everyday practicality and conceptual reasoning in explaining arithmetical techniques.

3.2 *Numerology* Excerpts from Ibn Ezra's *Sefer Haʿolam (Book of the World*, *ca* 12th century) and Levi Ben Gershon's *Maʿase Ḥoshev* (*The Art of the Calculator*,1321) offer examples of how Jewish scholars discussed combinatorial reasoning. Ben Gershon's analysis of various paired numbers and their connection to harmonic tones in music, skillfully described in his work *On Harmonic Numbers* commissioned by the French composer Phillipe de Vitry, is included in comprehensive detail [277–283]. With selections from Qalonymos Ben Qalonymos' *Sefer Melakhim* (*Book of Kings*, *ca* 14th century), Don Benveniste ben Lavi's *Encyclopedia* (1395), and Aaron Ben Isaac's *Arithmetic* (*ca* 15th century), readers can learn about "amicable numbers"—a pair of integers where the sum of the divisors of one number equals the other. These discussions are thought to be based on Thābit ibn Qurra's theorem (and proof) of finding amicable numbers from around the ninth century.

3.3 *Measurement and practical geometry* The passages from the 11th-century Jewish scholar Abraham Bar Ḥiyya's *Ḥibur Hameshiḥa Vehatishboret* (*The Treatise on Measuring Areas and Volumes*), an influential Hebrew work that was (partly) translated into Latin by Plato of Tivoli in 1145, show how Bar Ḥiyya's introduction to abstract geometry essentially resembles a manual on mensuration. Its purpose is more aligned with the Arabic tradition of *muʿāmalāt* (rulings governing commercial transactions) than with scholarly geometrical expositions. The method of heuristic reasoning (*heqesh taḥbuli*)—an iterative application of the classical method of "double false position"—applied by Levi Ben Gershon in calculating the Sine[2] of the fourth part of a degree as a minimum interval of his Sine table and described in his *Astronomy*[3] is particularly interesting [320–322].

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[1] See, e.g., pp. 228–229 on the multiplication sphere.

[2] The sine (Latin *sinus*) is a trigonometric function of an arc-angle. Capitalized "Sines" represent rescaled sine values for a non-unitary maximum sine (*sinus totus*).

[3] This is book 5 of part 1 of his major religious work, *Sefer Milḥamot HaShem* (*Wars of the Lord*).

3.4 *Scholarly geometry* The detailed English translations of Ben Gershon's commentary on Euclid's parallel postulate [*Elem.* 1.post. 5], especially of Ben Gershon's proof of this postulate by invoking his own more "self-evident" postulates, provide a remarkable introduction to the scholarship of medieval Jewish authors on conceptual geometry. The excerpt from his *Treatise on Geometry* (written after his larger full commentary on books 1–5 of Euclid's *Elements*, *ca post* 1337) shows his attempt to construct geometry on foundations stronger than Euclid's own work. The passages from *Sefer Meyasher ʿAqov* (*Book of the Rectifying of the Curved*) by Abner of Burgos—*alias* Alfonso di Valladolid, 1270–1348)—show Alfonso's treatment of the quadrature of the lune and the conchoid of Nicomedes. The editorial commentary about Alfonso's applications of the conchoid, an application that is different from parallels known in Greek sources, as well as its recognition in the West in the 14th century earlier than hitherto believed, will offer historians of mathematics a fascinating read [347–353].

3.5 *Algebra* Passages from Ben Gershon's *The Art of the Calculator*, an anonymously authored work from around 1200, and Elijah Mizraḥi's *Book of Number* illustrate various procedures for solving quadratic problems without using explicitly algebraic methods. Two unknown quantities are arithmetically derived from known values of their product and their sum (or difference). Ibn al-Aḥdab's Hebrew translation *Igeret Hamispar* (*The Epistle of the Number*, *ca* 15th century) of the Arabic mathematical text *Talkhīṣ aʿmāl al-ḥisāb* (*A Summary Account of the Operations of Computation*) written by Aḥmad ibn al-Bannāʾ (1256–1321) describes the arithmetical procedure of "double false position" (or the method of scales). In fact, al-Aḥdab's translation is the first Hebrew text to include explicitly algebraic terms (e.g., roots or "things", squares or "estates") along with their rules for multiplication and division [367–374].

In summary, the chapter on medieval Jewish mathematics offers historians unfamiliar with the Jewish (Hebrew) tradition a fascinating review of its ingenuity. For example, the method of solving plane triangles, a technique important for astronomical problems, in Ben Gershon's *Astronomy* is believed to be one of the earliest European accounts of the Islamic trigonometric solutions of planar triangles. In providing English translations based on primary Hebrew sources, often for the first time, the contributors of this chapter certainly warrant commendation from professional historians who are now able to reference this work in their own research.

4. Mathematics in the Islamic world in medieval Spain and North Africa

The third and last chapter of this sourcebook discusses the mathematical contributions made by Islamic scholars from medieval Spain and North Africa. The chapter begins by succinctly tracing the development of mathematics as successive rulers came to power in Muslim Iberia (*al-ʾAndalus*) during the Middle Ages. Subsequently, it outlines the mathematical activities in North Africa (*al-Maghrīb*) during this same period. The introductory essay contains useful pieces of information for readers wishing to see how mathematical ideas transferred between cultures. For example, in the introduction to chapter 3 we find:

> Al-Haṣṣār [*c*. twelfth century] wrote at least two mathematical works. One, the *Book of Demonstration and Recollection*, dealt with calculation and is the first book known using the horizontal bar to separate the numerator from the denominator of fractions. This practice spread rapidly in mathematical teaching in the Maghrib, where Fibonacci (who died sometime after 1240) learned of it and used it in his famous *Liber abbaci*. The book was also translated into Hebrew in the thirteenth century by Moses ibn Tibbon. [385]

Like the previous chapters, this chapter also categorizes the excerpts on medieval Islamic mathematics from al-Andalus and the Maghreb into topical sections:

- Arithmetic
- Algebra
- Combinatorics
- Geometry, and
- Trigonometry.

4.1 *Arithmetic* The excerpts from *Al-talkhīṣ aʿmāl al-ḥisāb* (*A Summary Account of the Operations of Computation*) and *Raf ʿal-ḥijāb ʿan wujūh aʿmāl al-ḥisāb* (*Raising the Veil on the Various Procedures of Calculation*) by Aḥmad ibn al-Bannāʾ (1256–1321) show his exposition of whole numbers, addition, subtraction, multiplication, division, and fractions in detail. Common arithmetical themes—for instance, the different orders (*martaba*) of numbers—are brought together from both these works to indicate as well his philosophical and mathematical reflections [387–388]. The excerpts from *Removing the Veil from the Science of Calculation* by 15th-century Maghrebi mathematician ʿAlī bin Muḥammad al-Qalaṣādī reveal how his account of arithmetical procedures differs from that of Ibn al-Bannāʾ. For instance, on the topic of multiplication [400–402], we see the contrast between al-Qalaṣādī's multiplicative method of "finding unknowns from knowns" and Ibn al-Bannāʾ's method of explaining multiplication in terms of addition.

4.2 *Algebra* In reading the excerpts from Ibn al-Bannāʾ's *Al-talkhīṣ aʿmāl al-ḥisāb* and *Raf ʿal-ḥijāb ʿan wujūh aʿmāl al-ḥisāb*, as well as those from his *Al-ʾuṣūlwa al-muqaddimāt fī al-jabr wa al-muqābala* (*Book on Fundamentals and Preliminaries for Algebra*, *ca* late 13th century), readers can see how Ibn al-Bannāʾ's explanation of the terms *al-jabr* "restoring" (adding to both sides of an equation the quantity subtracted from any one side) and *al-muqābala* "balancing" (subtracting terms among themselves so that the final equation does not contain the same terms on both sides) made these "techniques" integral to the calculations (*ḥisāb*) of unknown quantities. The operative rules for transacting with these *species*, i.e., numbers, unknowns, and squares, formed the arithmetic of algebra. These were then applied to solve various word problems in the two-part division of Ibn al-Bannāʾ's *Al-ʾuṣūl wa al-muqaddimāt fī al-jabr wa al-muqābala* [415–422].

4.3 *Combinatorics* Aḥmad ibn Munʿim's combinatorial problems from his *Fiqh al-Hiṣāb* (*On the Science of Calculation*, *ca* 13th century) are particularly interesting. In them, Ibn Munʿim discusses the combinations of possible words that can be formed from the 28 letters of the Arabic alphabet when specific rules of Arabic orthography are applied. The six problems from section 11 of Ibn Munʿim's book reveal how combinatorial mathematics was skillfully applied to semantic investigation [434–446]. Interested readers may compare this with the combinatorial expositions of the Indian mathematician Nārāyaṇa Paṇḍita, in his *Gaṇitakaumudī* (*Lotus Delight of Calculation*, 1356) [Kasuba and Plofker 2013, 55–61].

4.4 *Geometry* The excerpts from *On Measurement* of Abū ʿAbd Allah Muḥammad ibn ʿAbdūn (d. 976) reveal how geometrical shapes were actually measured, an activity of the *muhandis* "ones who measure". Interestingly, Ibn ʿAbdūn identifies the components of geometrical shapes that are *measurable*; his subsequent discussion on how each component is derived from the other shows what shapes meant to medieval Islamic geometers (or surveyors). The 11th-century geometer from medieval Spain, Al-Mut'taman ibn Hūd, the king of Saragossa (1081), blended Greek and Arabic geometry in his *Kitāb al-Istikmāl* (*Book of Perfection*). The passages on Ibn Hūd's proof of Heron's theorem [478–480], and his proof of Ceva's theorem using Menelaus' theorem almost 600 years before Giovanni Ceva proposed it in his *De lineis rectis* in 1678 [482–484], are especially insightful.

The selections from Muḥyī al-Dīn ibn al-Shukr al-Maghribī's *Recension of Euclid's* Elements (*ca* 13th century)—in particular, al-Maghribī's book 15, an adaptation of an unknown Arabic work on polyhedra—extended Hypsicles' treatment to all five regular solids [497–502]. In fact, a Hebrew translation of this anonymous Arabic work was made by the Jewish scholar Qalonymos ben Qalonymos in the 14th century. The section on scholarly geometry in the second chapter of this sourcebook includes excerpts from this Hebrew work [337–339].

4.5 *Trigonometry* Ibn Muʿādh's geometrical proof for determining the unknown lengths of arcs, their differences, and the ratio of their chord-lengths (or Sines), from his *Book of Unknowns of Arcs of the Sphere* (*ca.* 10th century) is an exemplary study of trigonometry in Muslim Spain during the Middle Ages. Ibn Muʿādh's exposition of the Sine theorem (and its application to a right triangle) as well as his method of solving spherical triangles, an important trigonometric result in determining the direction of Mecca, are particularly revealing of his scholarship [512–520]. Among later scholars, the passages on Jābir's Rule of Four Quantities from *Correction of the Almagest*, a 12th-century work by the influential Sevillian astronomer Abū Muḥammad Jābir ibn Aflaḥ, attest to the mastery Islamic scholars achieved in the study of spherical trigonometry during this period.

For attentive readers, this chapter brings to light several important and influential discoveries made by scholars from the Islamic West. Two in particular stand out for their mathematical elegance and conceptual geometry. The first is in the Cordovan astronomer Abū al-Qāsim ibn al-Samḥ's *The Plane Sections of a Cylinder and the Determination of Their Areas* (partially surviving in Qalonymos ben Qalonymos' Hebrew translation), where he proves the equivalence of the oblique section of a right circular cylinder and an ellipse constructed using the ordinary "gardener's method", i.e., moving a taut loop of string staked at two fixed ends, which he called a "triangle of movement" [457–468]. Incidentally, Ibn al-Samḥ also goes on to describe a method for measuring the area of this ellipse by relating its areal measure to that of circles inscribing and circumscribing the ellipse. The other remarkable excerpt is from Ibn Muʿādh's *On Twilight and the Rising of Clouds* where he offers geometrical arguments to ascertain the height of the Earth's atmosphere, a measure related to his analysis of what causes twilight [520–530].

5. Comments

The most valuable feature of this sourcebook is the frequent referencing of mathematical content (cited as excerpts of problems, solutions, or proofs) across different authors writing in different languages and in different timelines. For example, when reading about the Jewish scholar Bar Ḥiyya's method of dividing the area of a quadrilateral as found in his *Treatise on Measuring Areas and Volumes* [310], readers are made aware of its resemblance to the geometrical methods presented in an Arabic translation of Euclid's *On Divisions* by the 10th-century Persian geometer al-Sijzī [Hogendijk 1993, 143–162] as well as Fibonacci's methods described in the fourth chapter of his *Practical Geometry* [135–139]. Similarly, one comes to see the connection between medieval Islamic scholars (like al-Maghribī) and Jewish scholars (like Levi Ben Gershon) on their proposing "proofs" of Euclid's parallel postulate [495]. Such interconnections across traditions are extremely useful to professional historians tracing the development of a particular mathematical idea.

At the end of every chapter, the contributors have provided a list of primary and secondary sources from where the excerpted passages are cited. These lists, along with the chapter-bibliographies, make this sourcebook a credible research resource for historians of mathematics. There are, however, a few instances where references are lacking. For instance, in talking about the antiquity of treating quadratic problems, the contributors to the section in chapter 2 on algebra in Hebrew sources state:

> But even before the "official" algebra using the Khwārizmian terms (root/thing, square/property, cube, etc.) and six normal forms of linear and quadratic equations, quadratic problems were treated by methods that go back to Mesopotamia, namely, the reduction of problems to deriving the values of two unknown from their product and sum/difference with an implicit or explicit geometric model. [354]

Citing apposite references to studies on algebra in Mesopotamian sources—for example, Høyrup 2002 or Robson 2007, 102–127—would have helped interested readers pursue this assertion conveniently.

The layout of the original translations (mostly typeset in regular Arial font) followed by editorial commentaries (typeset in regular Times Roman font) is problematic in that it uses mixed typefaces and short first-line indentations that compress the textual content and strain readability. It is often the case with lengthy publications that good typography is compromised to meet the demands of printing. Such concessions, however, should be carefully considered given the time and money readers invest in the printed copy. Future editions of this sourcebook would benefit from a refreshed typography.

6. Conclusion

The exemplary scholarship made available through this sourcebook certainly towers over the minor shortcomings in its production. This sourcebook continues in the scholarly tradition of presenting excerpted translations of mathematical writings from the ancient and medieval worlds.[4] In this respect, it rises above its pedagogic or repertorial purposes to itself become the object of future studies in the historiography of mathematics.

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[4] See, for instance, Yăn and Shírán 1987, Clagett 1999, Datta and Singh 2001, Katz 2007, and Rashed and El-Bizri 2012.