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DETERRENCE OF THEFT IN A SITUATION OF COMPETITION BETWEEN FORMAL AND INFORMAL INSTITUTIONS

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Article abstract

The aim of this article is to highlight important factors neglected in modelling the effects of deterrent policies on thieves' activities in the real world. The methodology adopted consists of defining a world with three continuous spaces. Space in the centre has no institutions and no production. Only the other two spaces have institutions and are places for production. The study uses the Tullock contest fonction for n-players developed by Jia (2012) to identify thieves' efforts and the institutions' endowments at equilibrium. In contrast to the existing literature, our results indicate a perverse, indirect effect of institutions' deterrence strategies on thieves' activities and a negative effect of an increase in institutional deterrence on the total proportion of production stolen. This outcome therefore supports deterrence policies. Symmetric equilibrium becomes unstable when institutions have different production levels. However, we note that asymmetric equilibrium remains optimal, even in situations of differences in production across institutions. A confrontation between thieves from different areas can be a way for an institution to provide less deterrence in an asymmetric balance while guaranteeing a higher level of consumption than that under the opposing institution.

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DETERRENCE OF THEFT IN A SITUATION OF COMPETITION BETWEEN FORMAL AND INFORMAL INSTITUTIONS*

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RÉSUMÉ – L'objectif de cet article est de mettre en lumière des facteurs importants négligés dans la modélisation des effets des politiques de dissuasion sur les activités des voleurs dans le monde réel. La méthodologie adoptée consiste à définir un monde avec trois espaces continus. L'espace central n'a ni institutions ni production. Seuls les deux autres espaces ont des institutions et sont des lieux de production. L'étude utilise la fonction de concours Tullock pour n-joueurs développée par Jia (2012) pour identifier les efforts des voleurs et les dotations des institutions à l'équilibre. Contrairement à la littérature existante, nos résultats indiquent un effet pervers et indirect des stratégies de dissuasion des institutions sur les activités des voleurs et un effet négatif d'une augmentation de la dissuasion institutionnelle sur la proportion totale de la production volée. Ce résultat soutient donc les politiques de dissuasion. L'équilibre symétrique devient instable lorsque les institutions ont des niveaux de production différents. Cependant, nous constatons que l'équilibre asymétrique reste optimal, même dans des situations de différences de production entre les institutions. Une confrontation entre des voleurs de différentes régions peut être un moyen pour une institution de fournir moins de dissuasion dans un équilibre asymétrique tout en garantissant un niveau de consommation plus élevé que celui de l'institution adverse.

ABSTRACT – The aim of this article is to highlight important factors neglected in modelling the effects of deterrent policies on thieves' activities in the real world. The methodology adopted consists of defining a world with three continuous spaces. Space in the centre has no institutions and no production. Only the other two spaces have institutions and are places for production. The study uses the Tullock contest function for n-players developed by Jia (2012) to identify thieves' efforts and the institutions' endowments at equilibrium. In contrast to the existing literature, our results indicate a perverse, indirect effect of institutions' deterrence strategies on thieves' activities and a negative effect of an increase in institutional deterrence on the total proportion of production stolen. This outcome therefore

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supports deterrence policies. Symmetric equilibrium becomes unstable when institutions have different production levels. However, we note that asymmetric equilibrium remains optimal, even in situations of differences in production across institutions. A confrontation between thieves from different areas can be a way for an institution to provide less deterrence in an asymmetric balance while guaranteeing a higher level of consumption than that under the opposing institution.

Introduction

It is often argued that economists have only a limited understanding of the impact of deterrence policies on criminal activities in the real world, particularly due to the neglect of important factors in modelling (DiIulio, 1996). However, there have been some contributions that have considered these neglected elements. For example, Marceau (1997) examined the deterrence choices of jurisdictions and the localization of criminals by integrating the multi-jurisdictional dimension. However, his analysis was limited to symmetrical and internal equilibrium, which led Marceau and Mongrain (2002) to generalize the results. However, the rational elements of the theory of crime, as stated by Becker and taken up by Jaumard (2013), were only partially considered. Their analyses, inspired by the Hotelling model of spatial differentiation, were based on two identical jurisdictions located in two spaces, assuming that only the allocations of the courts are the subject of predation by criminals. This analysis integrates neither the socio-economic context nor the production. However, it is certain that the thief's effort influences the level of risk incurred, which is a probable cost different from the cost as such (Dubé, 2012; Luhmann, 2001).

Draca and Machin (2015) indicated that problems related to the specificity of crimes, specialization, reconversion, and indifference of the criminal, which have rarely been addressed in the literature, are likely to limit our understanding of the phenomenon, partly become some of the provisions in the modelling of deterrence are not considered. Thus, Marceau and Mongrain (2002), opting for a model without production but with identical endowments for the two localities, could not address the questions of technological differences and factor endowments. One study that considered this limit was that of Draca and Machin (2015); Draca *et al.* (2019), who examined the relationship between property prices and theft-related crime levels using data from the London Police Reporting System; this study concluded that different prices of goods play roles in the rate of crimes committed. Elasticities range from 0.3 to 0.4, and the authors indicated that the correlation between price change and crime rate is positive. However, these results remained empirical. Moreover, Angela *et al.* (2012) noted that economists have devoted more attention to the empirical verification of the model for the economic analysis of crime, including deterrence, from the perspective that the police can reduce crime by increasing the costs incurred by the criminal.

"Thefts and assaults put the citizens in a quite critical situation of bother and suffering, causing anxiety, mistrust and fear, which can make them wonder whether it is actually worthwhile to reflect on the theme of their control" (Cusson, 1983). According to North (1990), institutions are rules that govern the interactions between economic agents in a society. However, the operational inability of institutional structures remains a reality. In Burkina Faso, the local populations have established "Kogl-weogo" groups in rural areas to address security issues that the national security forces cannot adequately address (Ouedraogo, 2016). Friedman (1979) noted that examples of private deterrence have worked well in Ireland.

There are no Kogl-weogo groups in the western region of Burkina Faso. However, we can find a similar organization called the "Dozos", which plays the same role in terms of addressing security issues. These informal institutions use methods different from traditional methods, especially in terms of respect for human rights and formal institutions. When the members of these organizations catch a thief, if it makes sense, they flog him properly prior to handing him over to formal institutions that address him with respect for human rights. The Dozos in these areas operate in this way. In some cases, these informal institutions, for example, the Kogl-weogo groups of the Eastern and Central Plateau regions, might require the payment of compensation to the society: the return of the stolen property and the payment of a fine for their benefit. The Dozos arrest the bandits and cattle thieves and hand them over to the police. They are present in Burkina Faso, Mali, Ivory Coast and Guinea, and they are known as traditional Manding hunters, who also provide security for their populations. In terms of results, it seems that the thief is more dissuaded by the presence of informal institutions, such as Kogl-weogo and Dozos, in rural areas than by the presence of formal institutions, such as the police and gendarmerie. These differences in outcomes would be related to the different sanctioning methods used by the two types of formal and informal institutions. Since the establishment of Kogl-weogo in these rural areas, the complaints filed by the populations and the level of cattle theft have decreased significantly to the extent that, in areas where they are not present, many people will urge the authorizes to recognize the existence of these groups. Unlike a formal institution, access to which is difficult and costly, the permanent presence of informal institutions provides significant services to the population in terms of easy access and low costs. Conversely, the permanent presence of formal security forces generates enormous costs for the taxpayer, often with less significant results. The criminal in such a situation will wait for their departure to start operating.

Angela *et al.* (2012) argued that economists know little about the main factors identified in the economics of crime literature as key determinants of crime. The reason is that even hypotheses that appear to be consistent with US data for recent decades are inconsistent with data over longer horizons or across countries.

If we are unable to replicate the theoretical results in empirical studies, we must determine the reasons in the assumptions on which we have modelled these phenomena. We certainly have not considered the possible perverse effect of the modelling.

To make a contribution to this literature, based on the environment of the model proposed by Marceau and Mongrain (2002) and in light of the gaps mentioned, substantial changes are introduced. We relax the hypothesis of the identity of the courts (institution) and introduce two types of market and non-market production[1] into an environment with three spaces, i.e., three categories of thieves. Moreover, our modelling assumes competition between thieves of different institutions.

This article, which is an extension of the contribution of Marceau and Mongrain (2002), allows us to review a certain number of conclusions. Does this environment, which does not reflect reality, justify the result that the deterrent endowment has an ambiguous effect? What is the impact of an improvement in the model's environment on the main conclusions drawn by Marceau and Mongrain (2002)?

Examining how the presence of different types of economic activities can influence both criminal and law enforcement has been mostly overlooked. The interaction of law enforcement and criminal effort is also of first order importance.

This article diverges from the existing literature by assuming that there exists asymmetry between jurisdictions, both at the level of economic activity and in terms of the types of activities. Additionally, it considers criminal efforts.

This article is structured as follows. In section 1, an overview of the literature is provided. Section 2 presents the model and derives the behaviour of the thieves. In section 3, we study the problem of institutions by assuming that they have no coordination of actions, whereas in section 4, we examine the effectiveness of the derived equilibrium.

1. The Theories of Deterrence: Literature Review

Deutsch *et al.* (1987) studied the choices of the localization of criminals by considering exogenous levels of deterrence in different jurisdictions. Shavell (1991) studied the choice of the precautionary level that an individual exercises to protect his or her property in a context in which criminals can choose among several properties. The author showed that, in this case in which the precautionary level is observable, the equilibrium precautionary level can differ from the socially desirable level. The advantage of such modelling is that the author considered several properties. However, the existence of individuals in the other properties was not mentioned, suggesting that we are in a situation of unilateral deterrence instead of multidimensional deterrence. Zenou (2003) examined the spatial variations of crime, both between and within cities. He proposed two types of mechanisms: social interactions that stipulate that an individual is more likely to commit a crime if his or her peers commit one than if they do not commit crimes; and the distance to jobs, which indicates that remote residential location induces individuals

1. The need for such a distinction is provided in further development.

to commit more crimes. The model that Zenou proposed to examine the effect of the distance to jobs can be summarized as follows: all individuals are located in a line where all jobs are situated at the origin. For him, first, individuals must decide where to commit crimes. As stated above, individuals trade off between committing crimes where they reside and in the Central Business District (CBD). The underlying hypotheses of the model is that there is a greater protection in the CBD but poor protection in the residence. The main conclusion of the model is that all of the crimes will occur in the CBD. The study had the merit of considering two localities and two different jurisdictions.

However, the assumption that all jobs are concentrated in the CBD leads us to accept that there is no production in the places of residence; therefore, individuals will locate themselves in the CBD. However, the thieves will reside in the residence and will commit crimes in the CBD. Why not consider the existence of jobs also in a second place where individuals reside? Considering this criticism in the modelling could help to determine the complexity of the spatial crime analysis. Although Zenou (2003) did not model the crime decision, an environmental improvement of his model allows for analysing this aspect. He could add a second CBD on the other side of the residences and analyse the crime decisions. Our model considers this limit.

The theory of deterrence was articulated in its modern version in 1764 by Beccaria, who established his social contract according to which men gather and sacrifice part of their freedom in exchange for security. The freedom in question also integrates financial freedom in the sense that the resources that are allocated for the social contract are no longer for the realization of another social service, such as health or education. These material, human and financial resources, which ensure that the security offered is favourable to the development of economic activities, are grouped under the name of deterrent endowment. However, Beccaria's contribution, not comparing the social costs borne by the population or the benefits derived from this social contract, assumes that the benefits exceed the social costs, which can be considered a strong hypothesis. Another limitation of his analysis is that he was not interested in cost/benefit analysis on the part of the criminal.

One had to await the founding text of the economy of crime drafted by Becker in 1968 to see the cost/benefit analyses on the sides of the criminal and of society. Several investigations in various fields have attempted to verify this theory empirically. For Winter (2008), every dollar spent on a given deterrence policy is one dollar less that could be allocated to other policy options. It is therefore important to consider various options for deterrence policies when assessing the advantages and disadvantages of both empirical and theoretical analyses. The analysis of deterrence through theoretical modelling must therefore examine both direct and indirect effects.

Another recent contribution was that of Engel *et al.* (2015), who found a significant deterrent effect on participants who are averse to risk. Bayer *et al.* (2009) questioned the effect of the length of incarceration of the criminal on the continuation and misguidance of the thief in criminal activities. Indeed, Tonry (2008) noted that economist researchers tend to completely ignore the studies of deterrence conducted by non-economists. Labonte also pointed out two fundamental errors in econometric and economic investigations: "The economist does not ask himself whether punishment reduces crime or not. Rather, he asks, how much does the penalty reduce crime? Similarly, the economist is inclined to accept that penal sanctions deter" (Labonté, 2013).

The cost/benefit rationality on which Becker and the previous analyses were based was tested by the concept of risk rationality that was theoretically developed by Pires (2001).

Dubé (2012) even pointed out that the rationality of risk lies in a type of rationality that he called "other rationality", which is different from cost/benefit rationality and is based on the individual also being able to make decisions. Dube isolated the rationality of risk from extracts from qualitative interviews on serious crimes. According to this theory, the risk arises even if the criminal is not committing a crime; it puts the criminal in a very problematic situation, such as in the case of a wanted terrorist. However, this theory applies only to serious crimes and therefore does not in any way challenge Becker's contribution, particularly in the case of "non-serious crimes", as described in the case of this investigation.

2. The Model

We consider two non-identical institutions – more precisely, two structures in charge of the implementation of institutional rules – formal and informal, with non-identical endowments. The geography of model is shown in figure 1 in the annex. Bégin (2016) described these types of institutional guardianship structures, the roles of which are often poorly understood by the individuals themselves, leading to low-level equilibrium that is not desirable. As a reminder, institutional guardianship structures have as their mission to ensure prevention, surveillance, detection and, if necessary, repression (Charbonneau and Lachance, 2015).

Several forms of informal institutions in the field of security exist in Burkina Faso. The Kogl-weogo provide security in the Central Plateau region, in a similar way to the Dozos groups in the West. People intend to install Kogl-weogo cells in the West, which is against the Dozos' will. It happens that the Kogl-weogo constitute a traditional security force in the Central Plateau region, whereas the Dozos play the same traditional role in the West. This clarification is intended to avoid any confusion that could lead to inconvenience. The Dozos are already working in perfect harmony with the police and security forces. When there are problems, the Dozos inform the gendarmerie or the police. The Dozos are traditional Manding hunters, who also ensure the safety of their populations. They are present in Burkina Faso, Mali, Ivory Coast and Guinea. Regarding the Kogl-weogo, they were revealed to the public during Burkina Faso's transition in 2015. They are self-defence groups stemming from the precarious security situation that occurred in certain regions, such as the East and the Central Plateau. They hunt down and punish highway robbers and mostly cattle thieves. It has reportedly been stated that the first Kogl-weogo dates from the 1970s and had the mission of protecting reserved areas.

We consider three spaces: A, C, and B. Only two of the three – space A and space B – have formal and informal[2] institutions, respectively, and spaces A and B are separated by space C, which is considered a refuge zone for thieves. Unlike Marceau and Mongrain (2002), we assume the existence of production in the two spaces where there are institutions and an absence of production in the space without institutions. We consider the production of a commercial good in space B and a non-market service in space A, allowing for the testing of the impact of factorial and technological (and therefore price) endowment on crime. The results of Draca and Machin (2015) are theoretically checked. It is also assumed that thieves do not appropriate the endowments of institutions, as was the case in the analysis of Marceau and Mongrain (2002), but rather the productions within the institutions. The thieves, in the space without an institution, have the choice between operating in A or in B. Institution A uses its endowment to dissuade thieves from spaces A and C through dissuasive actions. Similarly, institution B uses its endowment to deter thieves from spaces B and C. Institutions therefore use their endowments to deter thieves whether they come from A, B or C.

The formal institutions have a more efficient technology but must pay a fixed cost. Indeed, the deterrent mechanisms of informal institutions are being established and are easily broken down in the field. These institutions even use mild and appropriate deterrents. Conversely, the deterrent mechanisms of formal institutions are difficult to eliminate in rural areas because they are quite heavy. For example, formal institutions need a place in the locality to store their deterrents and to rest. This place or area is no longer necessary in regard to informal institutions. Members of the informal deterrence team store their equipment elsewhere and rest in their homes. The formal institution returns to investment in deterrence. We call this investment in deterrence $T_A = T$ in the model, and the informal one is $T_B = \lambda T$, where $\lambda < 1$. In rural areas, we have small production levels and/or few criminals. The informal institution would be desirable because the fixed cost supported by the formal institution is sufficiently large.

2.1 *Elements of the model and justification of the presence of congestion*

The model developed is inspired by Weber's industrial location model (1909) and the Hotelling model developed by Hotelling (1929). Weber's model enables us to justify the location of the production of the marketable good and the non-market service, respectively, in the rural and urban areas. The absence of production in space C can be justified in part by the desert. Finally, the model is inspired by that of Marceau and Mongrain (2002).

2. In the remainder of the document, we use "institution" and "thief" instead of "jurisdiction" and "criminal".

Space C is considered to have a unit of measurement. Let us suppose that thief j is positioned in zone C. Considering the case in which the thief will act in A, as shown in the figure, the horizontal distance between thief j and the end of zone C on the side of zone A is given by d_{CA}^{j}. Similarly, the horizontal distance between the thief and the end of the side of zone B is $1-d_{CA}^{j}$. We assume that there is no distance for a thief who intends to intervene in the zone where he or she is, which is the case for thieves in zones A and B. In contrast, the thieves in space C must operate either in space A, where they traverse the distance d_{CA}^{j}, or in space B, where they traverse the distance $1-d_{CA}^{j}$. For a thief who is in C and will act in B, we must consider, respectively, the distances d_{CB}^{j} and $1-d_{CB}^{j}$. A distance $\overline{d}$ exists in space C in such a way that a thief in C is indifferent between operating in A or B. In addition to the distance to be travelled for certain thieves, the effort that they must exert in the expropriation of loot is considered. This element was ignored by Marceau and Mongrain (2002). Perhaps the authors assimilated it into the cost of distance. It is necessary, however, to distinguish between the cost of transport to be borne by certain thieves and the effort that they must exert at the time of the expropriation of the loot. The indifference of the thief noted at $\overline{d}$ is interpreted as a situation for which it becomes inactive in the game. To be more explicit, let us say that the thief, at this point, has a zero probability of being able to steal the good sufficiently in A and B. Conversely, thieves who are in C at a distance d_j less than $\overline{d}$ each have a probability equal to $\overline{d}$ to steal goods. Thieves in C who operate in B also have a probability of stealing equal to $1-\overline{d}$.

If he or she operates in A, thief i who is in A appropriates a relative proportion of the production denoted as α_A, which is the relative gain that the thief derives from his or her action. This proportion is a function of the deterrent endowment of the institution A denoted as T and the effort of thief i in space A denoted as e_A^i. We finally note $\alpha A\ (T, e_A^i)$ as the relative part of the production stolen by thief i in A. The part of the production stolen by thief i in A is equal to the probability of winning multiplied by the probability of playing (to steal). The probability of playing is equal to one because the thief is in the area of operation. The probability of winning is given by Tullock's function. Taking inspiration from models using the "Tullock contest type function" developed by Tullock (1980) and generalized by Jia (2012) and by Chowdhury and Sheremeta (2011) to analyse conflict situations and considering only what is stolen by a player positioned in A, we can denote the production stolen by thief i in A as $\alpha_A\ (T, e_A^i) = \frac{e_A^i}{\sum_{i=1} e_A^i + T}$.

In zone A, there are two types of theft. We have the theft committed by thieves positioned in A and that by the thieves positioned in C but operating in A. For each type of theft, we can list the players (thieves and institutions) who participate, allowing us to specify α_i. Contest functions are probabilistic choice functions that, to our knowledge, were first proposed by Luce (1959) to study individual choice. However, it was Jia (2012) who derived essentially the n-players version of the Tullock contest function. In this paper, it is the n-players version that is used.

We can note that this thief cannot operate in C – there is no take-home production and therefore no "rewards related to robbery indicated in the theory of crime", as stated by Jaumard (2013) – or in B because to operate in A would cost him or her less in terms of effort. The rationality of the thief would then lead him or her to give up such an initiative. To steal a proportion of production in a locality i noted (α_i), the thief must exert an effort. Similarly, the thief will face the institution that is supposed to protect the good and the people. If he or she operates in B, thief j who is in B appropriates a relative proportion of the production denoted as α_B, which is the relative gain that the thief derives from his or her action. This proportion is a function of the deterrent endowment of institution B denoted as $T_B = \lambda T$ with $\lambda < 1$ and the effort of thief i in space B denoted as e_B^i. We finally note $\alpha_B\ (\lambda T, e_B^i)$ as the part of the production stolen by thief i in B. The part of the production stolen by thief i in B is equal to the probability of winning multiplied by the probability of playing. The probability of playing is one because the thief is in the area of operation. The probability of winning is given by the Tullock function. Using "the Tullock contest type function" and considering only what a thief positioned in A steals, one can also write $\alpha_B\ (\lambda T, e_B^i) = \frac{e_B^i}{\sum_{i=1} e_B^i + \lambda T}$.

Finally, thief j in C can operate in A or B. If the thief operates in A starting from C, he or she appropriates a relative proportion of the production denoted as $\alpha_C A$. The part of the production stolen by thief j in C who operates in A is equal to the probability of winning multiplied by the probability of playing. The probability of playing is $\overline{d}$ because the thief is not in the area of the institution. The probability of winning is given by the Tullock function. This proportion is a function of the deterrent endowment T, the effort of thief j starting from C and operating in A denoted as e_{CA}^j and the distance d_{CA}^j that separates thief j from the boundary of space A. The distance d_{CA}^j is only an argument of the effort e_{CA}^j.

We note $\alpha_{CA}\ (T, e_{CA}^j, \overline{d})$ as the part of the production stolen by thief j starting from C and operating in A. As we indicated above, $\overline{d}$ is the probability that the thief will play. This probability will make it possible to calculate the expected gain later. Using "the Tullock contest type function" and noting that d_j is an argument of effort, one can write

$$\alpha_{CA}(T, e_{CA}^j, \overline{d}) = \overline{d} \frac{e_{CA}^j}{\sum_j e_{CA}^j + T}$$

We consider p_{CA}^j to be the probability that thief j will win with effort e_{CA}^j starting from C and operating in A. We can write $p_{CA}^j = \frac{e_{CA}^j}{\sum_j e_{CA}^j + T}$, and α_{CA} become:

$$\alpha_{CA}(T_A, e_{CA}^j, \overline{d}) = \overline{d} p_{CA}^j$$

If, in contrast, the thief operates in B starting from C, he or she appropriates a relative proportion of production denoted as α_{CB}, which is a function of the deterrent endowment λT of the effort of thief j starting from C and operating in B denoted e^j_{CB} and the distance d^j_{CB} that separates thief j from the boundary of space B. We finally note α_{CB} $(\lambda T, e^j_{CB}, 1-\overline{d})$ as the proportion of production stolen by thief j if he or she operates in B starting from C with indifference point $\overline{d}$; therefore, the probability of playing is equal to $1-\overline{d}$. Using the same reasoning as in the previous case, we note:

$$\begin{aligned}\alpha_{CB}(\lambda T, e^j_{CB}, 1-\overline{d}) &= (1-\overline{d})\frac{e^j_{CB}}{\sum_j e^j_{CB}+\lambda T}\\ &= (1-\overline{d})p^j_{CB},\end{aligned}$$

with $1-\overline{d}$ the probability of playing and p^j_{CB} the probability that thief j in C who operates in B will win.

In a paper entitled "Competition in law enforcement and capital allocation" in which production is considered, Marceau and Mongrain (2011) modelled the probability of criminals being arrested, and it is the same probability for all criminals in the region, whether they are close to the jurisdiction or not. Our paper is inspired by this type of modelling. However, we rather model the probability that criminals in C will choose to commit a crime in a given institution (region). While the probability of arrests of criminals in the Marceau and Mongrain (2011) model is a growing function of institutional staffing, in our model, the probability of committing a crime is a decreasing function of institutional staffing. In this paper, we do not rely on the notion of fixed uniform cost or fixed heterogeneous cost developed by Melitz (2003) in his business analysis model.

There are two possible ways to model the game between the institution and the criminal: (i) with congestion between criminals or (ii) without congestion. By referring to Blouin (2018), the first strategy is adopted in this paper. In our model, a criminal residing in A imposes congestion on other criminals who reside in A and criminals coming from C. The share of the production stolen by a criminal residing in A and a criminal coming from C are $\frac{e^i_A}{\sum_{i=1} e^i_A+T}$ and $\overline{d}\frac{e^j_{CA}}{\sum_j e^j_{CA}+T}$, respectively, with $\overline{d}=\overline{d}(T,\lambda T, e^i_A, e^i_B)$. The first term $\left(\frac{e^i_A}{\sum_{i=1} e^i_A+T}\right)$ would imply that criminals residing in A impose congestion on criminals residing in A. The second term $\left(\overline{d}\frac{e^j_{CA}}{\sum_j e^j_{CA}+T}\right)$ would imply that criminals residing in A also impose congestion on criminals coming from C. Indeed, $\overline{d}$ decreases with an increase of e^i_A, leading to a decrease in the stolen portion by the criminal coming from C. Thus, there is congestion between criminals residing in A and those coming from C. Moreover, a criminal residing in B imposes congestion on other criminals residing in B and on the criminals coming from C. The share of the production stolen by a criminal residing in B and a criminal coming from C are $\frac{e^i_B}{\sum_{i=1} e^i_B+\lambda T}$ and $(1-\overline{d})\frac{e^j_{CB}}{\sum_j e^j_{CB}+\lambda T}$, respectively, with $\overline{d} = \overline{d}(T, \lambda T, e_A^i, e_B^i)$. The first term $\left(\frac{e_B^i}{\sum_{i=1} e_B^i + \lambda T}\right)$ would imply that criminals residing in B also impose congestion on criminals residing in B. The second term $\left((1-\overline{d})\frac{e_{CB}^j}{\sum_j e_{CB}^j + \lambda T}\right)$ would imply that criminals residing in B also impose congestion on criminals coming from C. Indeed, $\overline{d}$ increases with an increase of e_B^i. When $\overline{d}$ increases, it equals a decrease of $1-\overline{d}$, leading to a decrease in the stolen portion by the criminal coming from C. Thus, there is congestion between criminals residing in B and those coming from C. The criminal in the region creates congestion on other thieves, whether residing in the region or coming from C. Our model considers this situation, which is why parameter $\overline{d}$ plays a very important role in the model. Recently, Blouin (2018) presented a theoretical model of conflict between two players with an intervention by a peacekeeping force considering congestion.

Some secondary factors, such as the economic and social context, make it possible to explain these specifications (Bégin, 2016). We have $\alpha_A(T, e_A^i) < 1$, which is the proportion of the production of the service stolen by a thief who is in space A, exert a force e_A^i.

$\alpha_{CA}(T, e_{CA}^j, \overline{d})$ is the proportion of the production of the service stolen by a thief who is in space C and has operated in space A. It is assumed that an increase in the effort deployed by the thief will lead to an increase in the proportion of the service stolen. However, the performance of the effort is decreasing. Formally, we have:

$$\frac{\partial \alpha_A(T,e_A^i)}{\partial e_A^i} = \alpha_{A2}(T, e_A^i) = \frac{(\sum_{i=1} e_A^i) + T - e_A^i}{(\sum_{i=1} e_A^i + T)^2} > 0;$$

$$\frac{\partial \alpha_{CA}(T,e_{CA}^j,\overline{d})}{\partial e_{CA}^j} = \alpha_{CA2}(T, e_{CA}^j, \overline{d}) = \overline{d}\frac{(\sum_{j=1} e_{CA}^j) + T - e_{CA}^j}{(\sum_{j=1} e_{CA}^j + T)^2} > 0;$$

$$\frac{\partial \alpha_B(\lambda T,e_B^i)}{\partial e_B^i} = \alpha_{B2}(\lambda T, e_B^i) = (1-\overline{d})\frac{(\sum_{i=1} e_B^i) + \lambda T - e_B^i}{(\sum_{i=1} e_B^i + \lambda T)^2} > 0;$$

$$\frac{\partial \alpha_{CB}(\lambda T,e_{CB}^j,1-\overline{d})}{\partial e_{CB}^j} = \alpha_{CB2}(\lambda T, e_{CB}^j, 1-\overline{d}) = (1-\overline{d})\frac{(\sum_{j=1} e_{CB}^j) + \lambda T - e_{CB}^j}{(\sum_{j=1} e_{CB}^j + \lambda T)^2} > 0.$$

Similarly, an increase in the endowment granted to an institution is synonymous with a decrease in the proportion of production stolen by the thief.

Formally, we have:

$$\frac{\partial \alpha_A(T,\, e_A^i)}{\partial T} = \alpha_{A1}\left(T,\, e_A^i\right) = \frac{-e_A^i}{(\sum_{i=1} e_A^i + T)^2} < 0;$$

$$\frac{\partial \alpha_B\left(\lambda T,\, e_B^i\right)}{\partial \lambda} = \frac{-Te_B^i}{(\sum_{i=1} e_B^i + \lambda T)^2} < 0 \text{ and}$$

$$\frac{\partial \alpha_B\left(\lambda T,\, e_B^i\right)}{T\partial \lambda} = \frac{-e_B^i}{(\sum_{i=1} e_B^i + \lambda T)^2} = \alpha_{B1}\left(\lambda T,\, e_B^i\right) < 0$$

$$\frac{\partial \alpha_{CB}\left(\lambda T, e^{j}_{CB},\, 1-\bar{d}\right)}{\partial \lambda} = (1-\bar{d})\frac{-Te^{j}_{CB}}{\left(\Sigma_{j=1} e^{j}_{CB}+\lambda T\right)^{2}} < 0;$$

$$\frac{\partial \alpha_{CB}\left(\lambda T, e^{j}_{CB},\, 1-\bar{d}\right)}{T\partial \lambda} = \left(1-\bar{d}\right)\frac{-e^{j}_{CB}}{\left(\Sigma_{j=1} e^{j}_{CB}+\lambda T\right)^{2}} = \alpha_{CB1}\left(\lambda T, e^{j}_{CB},\, 1-\bar{d}\right) < 0;$$

$$\frac{\alpha_{CA}\left(T,\; e^{j}_{CA},\, d_j\right)}{\partial T} = \alpha_{CA1}\left(T,\; e^{j}_{CA},\, d_j\right) = (\bar{d})\frac{-e^{j}_{CA}}{\left(\Sigma_{i=1} e^{i}_{A}+T\right)^{2}} < 0.$$

When the thief exerts zero effort, the proportion of production stolen is also zero.

$$\alpha_{CA}\left(T, 0, \bar{d}\right) = 0,\ \alpha_{A}\left(T,\ 0\right) = 0;$$

$$\alpha_{CB}\left(\lambda T,\ 0,\ 1-\bar{d}\right) = 0 \ \text{ and } \alpha_{B}\left(\lambda T, 0\right) = 0.$$

2.2 *Sequences of Events*

Each institution receives, from the public authority or the locality to which it belongs, an endowment for deterrence T_i. This endowment consists of men, materials and financial means. The thief, being in A, B or C, observes the level of endowment, the level of production Y_i in each space, and the efforts of the other thieves – the thief in C observes[3] the efforts e^{i}_{A} and e^{i}_{B}, the thief in A observes e^{j}_{CA} and the thief in B observes e^{j}_{CB} – and chooses the institution in which he or she will operate. Thereafter, he or she commits or moves to commit the theft and cashes the net gain. Let β_A and β_B represent the total proportions stolen in A and B. The remaining share for institution A after the committed theft is $(1-\beta_A)Y_A - T)$. Similarly, what remains for institution B is $(1-\beta_B)Y_B - \lambda\ T)$.

2.3 *Choices of Thieves*

We can determine a position in space C of equation $d = \bar{d}$ in such a way that the thief located there is indifferent to operating in A or B. All of the thieves on the left of this point will operate in A, and all of the thieves on the right will operate in B. When the thief from C finishes operating in A or B, he or she must return to his or her place of residence. We can therefore consider in this model that both the one-way and return trips made by the thief are costly in terms of resources. However, the outward journey can be made by requesting the services of a carrier, whose unit cost is t, and the distance travelled is d_j. In contrast, the return of the thief to his or her place of residence is more delicate. Indeed, the latter must carry the loot and avoid being spotted easily, since he or she prefers not to use the first type of means of transport. He or she decides to walk, implying a cost in terms of physical effort provided that can be evaluated in monetary terms. We assume by simplification of the model that the effort e^{j} currently used by the thief is the same as at the time of the robbery and would cost him or her the price. In summary,

3. The observation of the thief is made indirectly through the amount of stolen loot.

the cost of transport is $t\left(e^{j}+d_{j}\right)$ for the thief in C. For the other thieves in A and B, there is no distance or effort of return since they are all residents of A or B. Considering that the cost of transport and effort units is identical and equal, the value of $\bar{d}$ is determined by the following relation:

$$\begin{aligned}&\left[\alpha_{CA}\left(T, e_{CA}^{j}, \bar{d}\right)\right] Y_A-\left(\bar{d}+e_{CA}^{j}\right) t \\ &=\left[\alpha_{CB}\left(\lambda T, e_{CB}^{j}, 1-\bar{d}\right)\right] Y_B-\left(1-\bar{d}+e_{CB}^{j}\right) t\end{aligned} \tag{1}$$

This relation indicates that the mean net gain expected by the thief positioned in $\bar{d}$ in space A is equal to his or her average net gain expected in space B. The equation indicates that the net proportion of production stolen in A by a thief in C acting in A is equal to the net proportion of production stolen in B by a thief in C operating in B. The solution to this equation is given by $\bar{d}(T,\ \lambda\ T,\ e_A^i,\ e_B^i)$. Indeed, the indifference of the players in C with respect to spaces A and B can be only a function of the existing variables (acting forces) in A and B. In this specific case, it is indeed $T,\ \lambda\ T,\ e_A^i,\ e_B^i$. This result indicates that the distance $\bar{d}$, rendering the thief in C indifferent between institutions A and B, is a function of the deterrent endowments in A and B and the efforts of thief i in A and B. This result[4] differs from that of Marceau and Mongrain (2002) by considering the expropriation efforts e_A^i, e_B^i of thieves, and it can allow for better understanding of the effect of institutional deterrent endowments in the context of competition between thieves.

Lemma 1: Everything else being equal, $\bar{d}(T,\ \lambda\ T,\ e_A^i,\ e_B^i)$ is decreasing in T and in e_A^i and increasing in $\lambda\ T$ and in e_B^i.

When the deterrent allocation of institution A is increased, it becomes less attractive for thief i to operate in space A in as much as he or she appropriates a smaller quantity of non-market output in A. The thieves who were positioned to the right of $d=\bar{d}$ are no longer indifferent between operating in A or B. They will tend to operate in B. The point $\bar{d}$, rendering the thief indifferent, moves to the left following an increase in the institution's deterrent endowment A. There will be fewer thieves operating in space A and more thieves operating in space B following an increase in the deterrent endowment of institution A. However, the increase in T acts negatively on the effort of the thief who operates in A, whether he or she comes from A or from C: $\frac{\partial e_A^i}{\partial T}<0;\frac{\partial e_{CA}^j}{\partial T}<0$. This time, a new displacement of point $\bar{d}$ to the right will be observed. The final effect will depend on both types of movements. If the magnitude of the first displacement prevails, there will be a negative effect of the increase in the endowment on the proportion of thieves operating in A and a decrease in the proportion of production stolen in A. Similar reasoning, applied to case B, shows that, following a rise in $\lambda\ T$, the point rendering the thief indifferent moves to the right. There will be fewer thieves

4. The proofs of Lemma 1 and of the propositions that do not have a direct proof in this text are found in the mathematical annex.

operating in B. However, the increase of $\lambda\ T$ has negative impact on the effort e^i_B of thief i in B. In this case, we observe a new displacement of the point $\bar{d}$ to the left, and the final effect is the result of the two types of movements. Marceau and Mongrain (1999) indicated that the ambiguous impact could be partly explained by the existence of multiple equilibria in crime analyses. Our result improves on the existing analyses in the literature and makes it possible to better understand that there is a perverse indirect effect in a strategy of deterrence when we consider a population of three types of thieves with different efforts.

Proposal 1

When the thief in A increases his or her effort, it becomes less interesting for a thief in space C to operate in A since he or she appropriates a smaller quantity of non-market production. Similarly, when the thief in B increases his or she effort, it becomes less interesting for a thief in space C to operate in B since he or she appropriates a small amount of market production. This result indicates that the proportion of thieves in localities of production does not matter. Conversely, the effort exerted by each thief in such localities is determinant in the choice of the location of the thief in C. The proof of proposition 1 follows from Lemma 1.

2.3.1 *Total Proportion of the Stolen Production and Effect of the Endowment of Institutions*

Proportion of non-market services stolen in A

Let us consider β_A the total proportion of production stolen in A. This value is the sum of the proportion of the production stolen by the thieves who were present in A and the proportion of the production stolen in A by the thieves who are in space C. We can note the existence of a value $\bar{d}$ that makes it possible for the thief to be indifferent between operating in space A or B. Each thief j positioned in C and acting in A has a probability $\bar{d}$ of playing. The proportion of production expected to be stolen by thief j is therefore given by equation (2).

$$\alpha_{CA}\left(T, e^j_{CA}, \bar{d}\right)) = \bar{d} p^j_{CA} \tag{2}$$

Considering a level of effort given for each thief and the level $\bar{d}$, the proportion of market production stolen in space A is given by equation (3).

$$\beta_A = \sum_{i=1}^{\cdot} \alpha_A\left(T, e^i_A\right) + \sum_{j=1}^{\cdot} \alpha_{CA}\left(T, e^j_{CA},\ \bar{d}\right) \tag{3}$$

We note that $\beta_A = \sum_{i=1}^{\cdot} \alpha_A\left(T, e^i_A\right) + \bar{d}(\sum_{j=1}^{\cdot} p^j_{CA})$ with $\bar{d} p^j_{CA}$ gives the expected proportion of production stolen by thief j in space A located at a distance d^j_{CA}. Thereby, $\bar{d}(\sum_{j=1}^{\cdot} p^j_{CA})$ represents the total proportion of production stolen by the thieves in C operating in A. We finally have $\beta_A(T, e^i_A, e^j_{CA}, \bar{d})$ with $e^i_A(T)$ and $e^j_{CA}(T, \bar{d})$ such that

$$\frac{\partial e_A^i}{\partial T} = e_{A1}^i < 0, \frac{\partial e_{CA}^j}{\partial T} = e_{CA1}^j < 0, \frac{\partial e_{CA}^j}{\partial \bar{d}} = e_{CA2}^j < 0, \frac{\partial e_A}{\partial T} = e_{A1} < 0.$$

We see that β_A is a function of the endowment T of institution A, the efforts e_A^i and e_{CA}^j of the thieves in A and C, respectively, who act in space A of the distance $\bar{d}$, the distance at which the thief is indifferent between operating in A or B. Notably, e_{CA}^j is a function of d_{CA}^j.

Proportion of merchant production stolen in B

Each thief j positioned in C and acting in B has a probability $1-\bar{d}$ of seizing the good. The proportion of expected production stolen by this thief is therefore given by:

$$\alpha_{CB}\left(\lambda T, e_{CB}^j,\ 1-\bar{d}\right) = \left(1-\bar{d}\right) p_{CB}^j, \text{ with } p_{CB}^j = \frac{e_{CB}^j}{\sum_j e_{CB}^j + \lambda T} \tag{4}$$

Let us consider β_B the total proportion of production stolen in B. On the basis of the previous case, we have:

$$\beta_B = \sum_{i=1}^{\cdot} \alpha_B\left(\lambda T,\ e_B^i\right) + \sum_{j=1}^{\cdot} \left(1-\bar{d}\right) p_{CB}^j \tag{5}$$

We note that $\alpha_{CB}\left(\lambda T, e_{CB}^j, 1-\bar{d}\right) = \left(1-\bar{d}\right) p_{CB}^j$ represents the expected proportion of production stolen by thief j in space B. Thereby, $\sum_{j=1}\left(1-\bar{d}\right)\ p_{CB}^j$ is the expected total proportion of production stolen by all of the thieves in C operating in B.

We note that β_B is a function of the endowment $\lambda\ T$ of institution B among the efforts e_B^i and e_{CB}^j of the thieves, respectively, in B and in C and who act in space B of the distance $\bar{d}$.

We finally have $\beta_B(\lambda\ T, e_B^i, e_{CB}^j, 1-\bar{d})$ with $e_B^i(\lambda\ T)$ and $e_{CB}^j(\lambda\ T, 1-\bar{d})$ such that

$$\frac{\partial e_B^i}{\partial T_B} = \frac{\partial e_B^i}{T\partial\lambda} = e_{B1}^i < 0, \frac{\partial e_{CB}^j}{\partial T_B} = \frac{\partial e_{CB}^j}{T\partial\lambda} = e_{CB1}^j < 0, \frac{\partial e_{CB}^j}{\partial \bar{d}} = e_{CB2}^j < 0,$$
$$\frac{\partial e_B}{\partial T_B} = \frac{\partial e_B}{T\partial\lambda} = e_{B1} < 0.$$

These results can be analysed in light of what is generally obtained in the classical literature. In terms of similarity, we note the distance $\bar{d}$ rendering the thief indifferent. However, it should be noted that, in this investigation, a clear distinction is made between the distance and the proportion of thieves. In terms of divergence, we can note that the effort of the thief in the modelling is considered. Institutional endowment is the deterrent element of our model, whereas in the classical literature, Marceau and Mongrain (1999) and Marceau and Mongrain (2002) considered it part of this endowment. Staffing can be seen as the material, human and financial resources granted to the institution to work towards compliance with institutional arrangements. The human resources are seized by the remuneration of the employees assigned to the institution. The financial means allow for the acquisition of fuel and the payment of mission expenses and current operations. Material means are means for displacement, weapons, and munitions. For durable equipment, only the value of depreciation is considered. It is more realistic to believe that the thief seeks to appropriate the production of economic agents and not the institutional endowment.

Effect of a rise in the endowment of the institution on the proportion of non-market production stolen in A

Starting from the expression of

$$\beta_A(T,\ e^i_A, e^j_{CA}, \bar{d}) = \sum_{i=1} \alpha_A\left(T,\ e^i_A\right) + \sum_{j=1} \alpha^j_{CA}\ \left(T,\ e^j_{CA}, \bar{d}\right)\ (3),$$

where $\sum_{j=1} \alpha^j_{CA}\ \left(T,\ e^j_{CA}, \bar{d}\right) = \bar{d}\ \sum_{j=1} \frac{e^j_{CB}}{\sum_j e^j_{CB} + \lambda T}$. To have a simplified expression, we can write

$$\beta_A = \sum_{i=1}^{\cdot} \alpha^i_A + \bar{d} \sum_{j=1}^{\cdot} p^j_{CA}\ , \text{ with } p^j_{CA}\ = \sum_{j=1}^{\cdot} \frac{e^j_{CB}}{\sum_j e^j_{CB} + \lambda T} \tag{6}$$

To make the presentation simple, we adopt a synthetic notation in the presentation of the derivatives. Thus, the index numbers in the derivatives refer to the position of the argument in a function. Let us consider the function $\bar{d}$. We know that $\bar{d} = \bar{d}(T,\ \lambda\ T,\ e^i_A,\ e^i_B)$. It will therefore be noted that $\frac{\partial \bar{d}(T,\ \lambda T,\ e^i_A,\ e^i_B)}{\partial T} = \bar{d}_1$; $\frac{\partial \bar{d}(T,\ \lambda T,\ e^i_A,\ e^i_B)}{T\partial\lambda} = \bar{d}_2$; $\frac{\partial \bar{d}(T,\ \lambda T,\ e^i_A,\ e^i_B)}{\partial e^i_A} = \bar{d}_3$, and in the same way, $\frac{\partial \bar{d}(T_A,\ T_B,\ e^i_A,\ e^i_B)}{\partial e^i_B} = \bar{d}_4$. Similarly, for derivatives relating to $\alpha^j_{CA}(T,\ e^j_{CA}, \bar{d})$, we note $\frac{\partial \alpha^j_{CA}(T,\ e^j_{CA}, \bar{d})}{\partial T} = \alpha^j_{CA1}$, $\frac{\partial \alpha^j_{CA}(T,\ e^j_{CA}, \bar{d})}{\partial e^j_{CA}} = \alpha^j_{CA2}$ and $\frac{\partial \alpha^j_{CA}(T,\ e^j_{CA}, \bar{d})}{\partial \bar{d}} = \alpha^j_{CA3}$. For the derivatives related to $\alpha^j_{CB}(\lambda\ T,\ e^j_{CB}, 1-\bar{d})$, we note $\frac{\partial \alpha^j_{CB}(\lambda T,\ e^j_{CB},\ 1-\bar{d})}{T\partial\lambda} = \alpha^j_{CB1}$, $\frac{\partial \alpha^j_{CB}(\lambda T,\ e^j_{CB}, 1-\ \bar{d})}{\partial e^j_{CB}} = \alpha^j_{CB2}$ and $\frac{\partial \alpha^j_{CB}(\lambda T,\ e^j_{CB}, 1-\bar{d})}{\partial \bar{d}} = \alpha^j_{CB3}$. We apply this notation in our derivations.

The derivative of β_A with respect to T is thus given by:

$$\frac{\partial \beta_A}{\partial T} = \sum_{i=1}^{\cdot} (\alpha^i_{A1} + \alpha^i_{A2} e^i_{A1}) + \bar{d}\left[\sum_{j=1}^{\cdot} p^j_{CA1} + \sum_{j=1}^{\cdot} p^j_{CA2} e^j_{CA1} + \sum_{j=1}^{\cdot} p^j_{CA2} e^j_{CA2} \bar{d}_1\right] + (\bar{d}_1 + \bar{d}_3 e^i_{A1})(\sum_{j=1}^{\cdot} p^j_{CA}) \tag{7}$$

When the deterrent institutional endowment increases, the thief's effort decreases, thus leading to a small proportion of stolen production. Indeed, the objective of the endowment is to dissuade the thief. When the thief integrates risk into his or her behaviour, he or she feels that it would be difficult to appropriate a large part of production in the face of a larger endowment. The sign of the derivative of β_A with respect to the institutional endowment T depends on the sign of the elements that constitute it. The first term $\sum_{i=1}^{\cdot} (\alpha^i_{A1} + \alpha^i_{A2} e^i_{A1})$ is negative because it is the sum of negative sign elements. The first sub-term $\sum_{i=1}^{\cdot} \alpha^i_{A1}$ measures the direct effect of an increase in deterrent endowment T on the proportion of non-market production stolen by thieves in space A. The second sub-term $\sum_{i=1}^{\cdot} \alpha^i_{A2} e^i_{A1}$ indicates the indirect effect by way of the thief's effort to increase the deterrent endowment T on the proportion stolen by the thieves who are in A. The second term $\bar{d}\sum_{j=1}^{\cdot} p^j_{CA1}$ measures the direct effect of an increase in the deterrent endowment T on the proportion of non-market production stolen by thieves in space C and operating in A. This term is negative. The third term $\bar{d}\sum_{j=1}^{\cdot} p^j_{CA2} e^j_{CA1}$ measures the indirect effect via effort of the increase in endowment T on the proportion of non-commercial production stolen by thieves in C operating in A. An increase in effort acts positively on the proportion of the stolen production of the agent, and an increase in the dissuasive endowment reduces the effort of the agent: the term is therefore negative.

The fourth term $\bar{d}\sum_{j=1}^{\cdot} p^j_{CA2} e^j_{CA2} \bar{d}_1$ measures the indirect effect via the line making the thief indifferent between A and B and the effort of the thief in C of an increase in Tof institution A. The increase in T reduces the probability of thieves in C operating in space A. The value of $\bar{d}$ decreases and thus translates into a shift from right to left and a decrease in the effort of the thief in C acting in A. However, e^j_{CA2} translates rather into a rise in the effort of the thief in C operating in A. Similarly, an increase in the effort increases the proportion of production stolen. From the above, we can deduce that the fourth term is negative.

The fifth term $(\bar{d}_1 + \bar{d}_3 e^i_{A1})(\sum_{j=1}^{\cdot} p^j_{CA})$ seems ambiguous. However, there are conditions for the effect to fulfil to avoid this ambiguity.

Conditions of an Unambiguous Effect of an Increase in the Institutional Endowment in A: They Come from the Constancy of $\bar{d}$. The conditions for $\frac{\partial \beta_A}{\partial T} < 0$ are derived from the seventh term analysed, which has an ambiguous effect. These direct and indirect effects of T occur via the straight line making the thief indifferent about the proportion of production stolen in A by thieves in C. Two forces are opposed in this situation: (i) on the one hand, the institution through endowment T can move the line point $\bar{d}$ to the left; and (ii) on the other hand, the effort of thief i in A, decreasing with the increase of T, can constitute an opportunity for the thieves in C to decide to act in A. This last effect causes $\bar{d}$ to move to the right. If the indirect effect prevails, the term will be positive and lead to an ambiguous effect of T on β_A. If, in contrast, the direct effect prevails, and then an increase in T reduces the proportion of production stolen in A.

The condition under which the effect of a deterrence endowment is not ambiguous is given by the equation

$$\left(\bar{d}_1 + \bar{d}_3 e^i_{A1}\right)\left(\sum_{j=1}^{\cdot} p^j_{CA}\right) \leq 0. \tag{8}$$

It suffices that

$$\bar{d}_1 + \bar{d}_3 e^i_{A1} \leq 0 \tag{9}$$

Finally, we have $\bar{d}_1 \leq -\bar{d}_3 e^i_{A1}$. In other words, we obtain $\frac{\partial \bar{d}}{\partial T} \leq -\frac{\partial \bar{d}}{\partial e^i_A}\frac{\partial e^i_A}{\partial T}$, which means that the reduction in the effort of the thief in A via the increase of the deterrent endowment must not lead to a return movement of $\bar{d}$ to the right at a magnitude greater than that of the displacement linked to the action of the thief of institution A. When we totally differentiate $\bar{d}$, we arrive at this condition. Thereby, the effect of an increase in the deterrence endowment on the proportion of production stolen is negative. It is therefore not ambiguous, as Marceau and Mongrain (2002) believed.

The effect of an increase in the institution's endowment on the proportion of the production stolen from a good merchant

We have

$$\beta_B(\lambda\, T,\, e^i_B, e^j_{CB}, 1-\bar{d}) = \sum_{i=1}^{\cdot} \alpha_B\left(\lambda T,\, e^i_B\right) + \sum_{j=1}^{\cdot} \alpha_{CB}\left(\lambda T,\, e^j_{CB}, 1-\bar{d}\right) \tag{10}$$

We can write:

$$\beta_B = \sum_{i=1}^{\cdot} \alpha^i_B + (1-\bar{d})\sum_{j=1}^{\cdot} p^j_{CB}, \text{ with } p^j_{CB} = \frac{e^j_{CB}}{\sum_j e^j_{CB} + \lambda T},$$

which leads to:

$$\begin{aligned}\frac{\partial \beta_B}{T\partial \lambda} = &\sum_{i=1}^{\cdot} (\alpha^i_{B1} + \alpha^i_{B2} e^i_{B1}) + (1-\bar{d}) \left[\sum_{j=1}^{\cdot} p^j_{CB1} + \sum_{j=1}^{\cdot} p^j_{CB2} e^j_{CB1} \right. \\ & \left. + \sum_{j=1}^{\cdot} p^j_{CB2} e^j_{CB2} \bar{d}_2 \right] - \left(\bar{d}_2 + \bar{d}_4 e^i_{B1} \right) \left(\sum_{j=1}^{\cdot} p^j_{CB} \right)\end{aligned} \tag{11}$$

As in case A, each of the first six terms is also negative. Only the last term $-\left(\bar{d}_2 + \bar{d}_4 e^i_{B1}\right)\left(\sum_{j=1}^{\cdot} p^j_{CB}\right)$ seems to have an ambiguous effect. However, there are conditions for it not to be ambiguous.

Conditions of an Unambiguous Effect of an Increase in the Institutional Endowment in B: They Come from the Constancy of $\bar{d}$. The condition of an unambiguous effect is as follows:

$$\bar{d}_2 \geq -\bar{d}_4 e^i_{B1} \tag{12}$$

In other words, we have $\frac{\partial \bar{d}}{T\partial \lambda} \geq -\frac{\partial \bar{d}}{\partial e^i_B}\frac{\partial e^i_B}{T\partial \lambda}$, which means that the reduction in the effort of the thief in B via the increase in the deterrent endowment in B must not lead to a return shift of $\bar{d}$ to the left at a magnitude greater than that of the displacement linked to the action of institution B.

When we totally differentiate $\bar{d}$, we reach this condition. We can therefore conclude that a deterrent endowment has a negative effect on the proportion of production stolen.

The effects of an increase in $\bar{d}$ on the proportions of stolen production

In institution A, $\beta_A = \sum_{i=1}^{\cdot} \alpha^i_A + (\bar{d}) \sum_{j=1}^{\cdot} p^j_{CA}$. The effect of $\bar{d}$ is given by:

$$\frac{\partial \beta_A}{\partial \bar{d}} = \sum_{j=1}^{\cdot} p^j_{CA} \tag{13}$$

Finally, we have:

$$\frac{\partial \beta_A}{\partial \bar{d}} = \sum_{j=1}^{\cdot} \frac{e^j_{CA}}{\sum_j e^j_{CA} + T} > 0 \tag{14}$$

The displacement of point $\bar{d}$ on the right increases the proportion of non-market production stolen in A.

$$\beta_B = \sum_{i=1}^{\cdot} \alpha^i_B + (1-\bar{d}) \sum_{j=1}^{\cdot} p^j_{CB}$$

Similarly, in institution B, the effect of an increase in $\bar{d}$ is given by:

$$\frac{\partial \beta_B}{\partial \bar{d}} = -\sum_{j=1}^{\cdot} p_{CB}^{j} \tag{15}$$

Finally, we have:

$$\frac{\partial \beta_B}{\partial \bar{d}} = -(\sum_{j=1}^{\cdot} \frac{e_{CB}^{j}}{\sum_j e_{CB}^{j} + \lambda T}) \tag{16}$$

We obtain

$$\frac{\partial \beta_B}{\partial \bar{d}} < 0 \tag{17}$$

A displacement of point $\bar{d}$ on the right decreases the proportion of market production stolen in B. These results show that the two institutions have an interest in cooperating to avoid the negative externalities of their respective actions. The increase in the institution's endowment produces two types of contradictory movements of the line that rendered the thief indifferent concerning operation in the two spaces. The first movement is the displacement of the point to the right of the institutional space, while the second movement affects the distance of this point from the institutional space. It is a movement of return from the point to its initial position due to the perverse effect of a decrease in the effort of the thief in the space where the institution is located. Thus, the reduction in the efforts of the thieves in A and B induces the thief in C to operate in the space of institution A and institution B. This result was not explained in previous analyses.

3. Deterrence between Institutions without Coordination and Equilibrium

3.1 *Problem of Institution A*

Institution A must maximize the net consumption of the population of its space, such as $(1-\beta_A)Y_A - T$, with $\beta_A = \sum_{i=1}^{\cdot} \alpha_A^i + \bar{d}\sum_{j=1}^{\cdot} p_{CA}^j$.

Also noting $\alpha_A^i = \alpha_A(T,\ e_A^i), p_{CA}^j = p_{CA}\left(T,\ e_{CA}^j\right)$, considering $e_A^i(T)$ and $e_{CA}^j(T, d_j)$.

The following problem is written:

$$\max_T \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_A^i + \bar{d}\sum_{j=1}^{\cdot} p_{CA}^j\right)\right] Y_A - T \tag{18}$$

The solution of this problem is characterized by the first-order condition:

$$-\left[\sum_{i=1}^{\cdot}(\alpha_{A1}^{i}+\alpha_{A2}^{i}e_{A1}^{i})+(\bar{d}_{1}+\bar{d}_{3}e_{A1}^{i})\left(\sum_{j=1}^{\cdot}p_{CA}^{j}\right)\right.$$
$$\left.+\bar{d}(\sum_{j=1}^{\cdot}(p_{CA1}^{j}+p_{CA2}^{j}e_{CA1}^{j}))\right]Y_{A}-1\leq 0 \tag{19}$$

This condition can be written as follows:

$$-\left[\sum_{i=1}^{\cdot}\alpha_{A2}^{i}e_{A1}^{i}+\bar{d}(\sum_{j=1}^{\cdot}p_{CA2}^{j}e_{CA1}^{j})\right]Y_{A}-\left[\sum_{i=1}^{\cdot}\alpha_{A1}^{i}+\bar{d}\left(\sum_{j=1}^{\cdot}p_{CA1}^{j}\right)\right]Y_{A}$$
$$-\left[(\bar{d}_{1})\left(\sum_{j=1}^{\cdot}p_{CA}^{j}\right)\right]Y_{A}-\left[(\bar{d}_{3}e_{A1}^{i})\left(\sum_{j=1}^{\cdot}p_{CA}^{j}\right)\right]Y_{A}-1\leq 0 \tag{20}$$

From equation (20), five groups of terms can be identified corresponding to five types of effects. Let us note that, in the classical literature, there were only three terms and therefore three types of effects. Thereby, our results represent a significant contribution.

Term 1 $-\left[\sum_{i=1}^{\cdot}\alpha_{A2}^{i}e_{A1}^{i}+\bar{d}(\sum_{j=1}^{\cdot}p_{CA2}^{j}e_{CA1}^{j})\right]Y_{A}$ represents the effect due to the reduction in the thieves' efforts in A and C as a result of an increase in institution A's endowment to non-market non-marketable production. This term has not been demonstrated in the classical literature and represents a limit of the existing results.

Term 2 $-\left[\sum_{i=1}^{\cdot}\alpha_{A1}^{i}+\bar{d}\left(\sum_{j=1}^{\cdot}p_{CA1}^{j}\right)\right]Y_{A}$ indicates that the greater that the deterrent effect is, the less that the proportion is of non-market production that each of the two types of thieves in A and C manages to steal. This term has already been mentioned in the literature (see Marceau and Mongrain (2002) as an example). However, it is more elaborate in the present investigation with the appearance of the share lost by each group of thieves, introducing a slight difference compared to the classical results.

Term 3 $-\left[(\bar{d}_{1})\left(\sum_{j=1}^{\cdot}p_{CA}^{j}\right)\right]Y_{A}$ indicates the displacement towards the left of the point making the thief in C indifferent to operating in A or B; as a result of an increase in the deterrent endowment, this term reduces the proportion of non-market production stolen by thieves in C operating in A.

Term 4 $\left[(\bar{d}_{3}e_{A1}^{i})\left(\sum_{j=1}^{\cdot}p_{CA}^{j}\right)\right]Y_{A}$ indicates that the rightward movement of the point making the thief in C indifferent to operating in A or B, as a result of a reduction in the thief's effort in A via the increase in the deterrent endowment, reduces the proportion of non-market production stolen by thieves in C operating in A.

Term 5, namely -1, is well known in the literature. It indicates the sacrifices that the population must make to finance the deterrent actions in their space. The greater that the staffing effort is for deterrence, the greater that the sacrifice made is in terms of consumption by the population. The consumption of space is equal to the production minus the institutional expenses and the quantity of stolen production. The idea is that the endowment for deterrence is financed by levies at the level of quantity produced.

3.2 *Problem of Institution B*

Institution B must maximize the net consumption of the population of its space as follows:

$$(1-\beta_B)Y_B - \lambda\ T, with \beta_B = \sum_{i=1}^{\cdot} \alpha_B^i + (1-\bar{d}) \sum_{j=1}^{\cdot} p_{CB}^j$$

Also noting $\alpha_B^i = \alpha_B(\lambda\ T,\ e_B^i)$, $(1-\bar{d})p_{CB}^j = \alpha_{CB}\left(\lambda T,\ e_{CB}^j\right)$ and considering $e_B^i\,(\lambda T)$ and $e_{CB}^j(\lambda\ T, d_j)$, we can write:

$$\max_{\lambda T} \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_B^i + (1-\bar{d}) \sum_{j=1}^{\cdot} p_{CB}^j\right)\right] Y_B - \lambda T \tag{21}$$

The solution of this problem is given by the first-order condition:

$$\begin{aligned} -\Bigg[&\sum_{i=1}^{\cdot} (\alpha_{B1}^i + \alpha_{B2}^i e_{B1}^i) + \left(\bar{d}_2 + \bar{d}_4 e_{B1}^i\right) \left(\sum_{j=1}^{\cdot} p_{CB}^j\right) \\ &+ (1-\bar{d})(\sum_{j=1}^{\cdot} (p_{CB1}^j + p_{CB2}^j e_{CB1}^j))\Bigg] Y_B \ -1 \leq 0 \end{aligned} \tag{22}$$

It can be written as follows:

$$\begin{aligned} &-\left[\sum_{i=1}^{\cdot} \alpha_{B2}^i e_{B1}^i + (1-\bar{d})(\sum_{j=1}^{\cdot} p_{CB2}^j e_{CB1}^j)\right] Y_B \\ &\quad -\left[\sum_{i=1}^{\cdot} (\alpha_{B1}^i) + (1-\bar{d}) \left(\sum_{j=1}^{\cdot} p_{CB1}^j\right)\right] Y_B - \left[(\bar{d}_2)\left(\sum_{j=1}^{\cdot} p_{CB}^j\right)\right] Y_B \\ &\quad -\left[(\bar{d}_4 e_{B1}^i)\left(\sum_{j=1}^{\cdot} p_{CB}^j\right)\right] Y_B - 1 \leq 0 \end{aligned} \tag{23}$$

The term $-\left[\sum_{i=1}^{\cdot} \alpha_{B2}^{i} e_{B1}^{i} + (1-\bar{d})(\sum_{j=1}^{\cdot} p_{CB2}^{j} e_{CB1}^{j})\right] Y_B$ represents the effect due to the decrease in the efforts of the thieves in B and C as a result of an increase in the endowment of institution A on the level of non-stolen market production.

The term $-\left[\sum_{i=1}^{\cdot} (\alpha_{B1}^{i}) + (1-\bar{d}) \left(\sum_{j=1}^{\cdot} p_{CB1}^{j}\right)\right] Y_B$ indicates that the greater the deterrent effect is, the less the proportion is of the market production that each of the two types of thieves in B and C manages to steal.

The term $-\left[(\bar{d}_2)\left(\sum_{j=1}^{\cdot} p_{CB}^{j}\right)\right] Y_B$ indicates that the displacement to the right of the point making the thief in C indifferent to the idea of operating in A or B, along with an increase in deterrent endowment $\lambda\ T$, reduces the proportion of market production stolen by the thieves in C operating in B.

The term $-\left[(\bar{d}_4 e_{B1}^{i})\left(\sum_{j=1}^{\cdot} p_{CB}^{j}\right)\right] Y_B$ indicates that the displacement to the left of the point making the thief in C indifferent to operating in A or B, following a decrease in the thief's effort in B via the increase in the deterrent endowment, reduces the proportion of market production stolen by the thieves in C operating in B.

The term -1 indicates the sacrifices that the population must make to finance the deterrent actions in their space.

Proposal 2

Let us consider λ^* and e_B^* to find the solution to

$$-\left[(1-\bar{d})(\sum_{j=1}^{\cdot}(p_{CB1}^{j} + p_{CB2}^{j} e_{CB1}^{j}))\right] Y_B = 1 \tag{24}$$

with T^* and 0 as the solution to

$$\bar{d}(T^*, \lambda^*\ T^*,\ 0,\ e_B^*) = 0; \tag{25}$$

there is an equilibrium without coordination between the institutions with a non-zero deterrent endowment level $(T^*,\ \lambda^* T^*)$ and a level of effort of the thieves in A and B $(0,\ e_B^*)$ in such a way that all thieves in C operate in B, and no thief operates in A if the following conditions are fulfilled:

$$\text{(i)} \sum_{i=1}^{\cdot} \alpha_A^{i*}(T^*,\ 0) + \sum_{j=1} \alpha_{CA}^{j*}(T^*,\ 0,0) + \alpha_{A2}^{i*}(T^*,0) \geq 0 \tag{26}$$

$$\text{(ii)} \sum_{i=1}^{\cdot} \alpha_A^{i*}(T^*,\ 0) + \sum_{j=1} \alpha_{CA}^{j*}(T^*,\ 0,0) + \alpha_{CA2}^{i*}(T^*,0,0) \geq 0 \tag{27}$$

$$\text{(iii)} \sum_{i=1}^{\cdot} \alpha_B^{i*}\left(\lambda^* T^*,\ 1\right) + \sum_{j=1} \alpha_{CB}^{j*}\left(\lambda^* T^*,\ 1,1\right) + \alpha_{B2}^{i*}\left(\lambda^* T^*,1\right) \leq 0 \tag{28}$$

$$\text{(iv)} \sum_{i=1}^{\cdot} \alpha_B^{i*}\left(\lambda^* T^*,\ 1\right) + \sum_{j=1} \alpha_{CB}^{j*}\left(\lambda^* T^*,\ 1,1\right) + \alpha_{CB2}^{i*}\left(\lambda^* T^*,1\right) \leq 0 \tag{29}$$

$$\text{(v)} \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_B^{i*}\left(\lambda^* T^*, e_B^*\right) + \sum_{j=1} \alpha_{CB}^{j*}\left(\lambda^* T^*, e_B^*,\ 1\right)\right)\right] Y_B - \lambda^* T^* = Y_A - T^* \tag{30}$$

Conditions (i) and (ii) indicate that, when no thief provides a strictly positive effort in space A, if an additional thief, whether from A directly or from C, operated in institution A, the total proportion of production stolen under this institution would not decrease. This result shows that it is not the proportion of thieves in a space but rather the effort exerted by each thief that is important. The reduced form of conditions (i) and (ii) are, respectively,

$$\alpha_{A2}^{i*}\left(T^*,0\right) \geq 0 and\ \alpha_{CA2}^{j*}\left(T^*,0,0\right) \geq 0 \tag{31}$$

Condition (iii) indicates that when all thieves in B operate in institution B with maximum effort e=1 each, if a lesser thief operates in B, the total proportion of production stolen would increase, which is explained by the decreasing efficiency of the effort. Thus, there is an optimal effort beyond which any increase in effort is detrimental to the thief.

Condition (iv) indicates that, when all of the thieves in C are operating in B with maximum effort $e = 1$ each, if one fewer thief operates in B, the total proportion of production stolen will increase.

Condition (v) ensures that, since all thieves operating in B choose an optimal effort, B has no interest in taking the place of A by choosing $\lambda > 1$, indicating that $\lambda^* T^* > T^*$ can never happen.

In the next section, we study the equilibrium of the model.

4. Solving the Problem

To solve the model, we must specify the issue. Thus, we consider n_A and n_B the number of thieves in zones A and B, respectively. Similarly, we note n_{CA} and n_{CB} thieves coming from C and operating, respectively, in A and B. The functions of forces are specified as follows.

$$e_A^i = \frac{1}{T}; e_{CA}^j = \frac{1}{T} - d_{CA}^j;\ e_{CB}^j = \frac{1}{\lambda T} - d_{CB}^j$$

Note that these specifications are consistent with the assumptions that we made from the outset.

We derive the equilibrium values of the variables of the problem, which are the distance $\bar{d}^*$ rendering thieves indifferent, equilibrium deterrence endowments T^* and $\lambda^* T^*$, and the equilibrium efforts of thieves in zones A (e_A^*), B (e_B^*) and C $(e_{CB}^*; e_{CA}^*)$. We are noting $\sum X_{CA}^j = X_{\dot{C}A}$. For example, $\sum d_{CA}^j = d_{\dot{C}A}$ and $\sum d_{CB}^j = d_{\dot{C}B}$

4.1 *Equilibrium value of $\bar{d}$*

We consider again the problem of institutions A and B. For institution A, we have:

$$\max_T \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_A^i + \sum_{j=1}^{\cdot} \alpha_{CA}^j\right)\right] Y_A - T$$

Another expression of this problem is given by:

$$\max_T \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_A^i + \sum_{j=1}^{\cdot} \alpha_{CA}^j\right)\right] Y_A - T$$
$$= \max_T \left[1 - \left(\sum_{i=1}^{\cdot} \frac{e_A^i}{\sum e_A^i + T} + \bar{d} \sum_{j=1}^{\cdot} \frac{e_{CA}^j}{\sum e_{CA}^j + T}\right)\right] Y_A - T$$

Considering the expressions of the efforts and the number of thieves in each region and after algebraic adjustments, we have:

$$\max_T \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_A^i + \bar{d} \sum_{j=1}^{\cdot} \alpha_{CA}^j\right)\right] Y_A - T$$
$$= \max_T \left[1 - \left[\frac{n_A}{T^2 + n_A} + \bar{d} \frac{n_{CA} - d_{\dot{C}A} T}{n_{CA} - T d_{\dot{C}A} + T^2}\right]\right] Y_A - T$$

Deriving this equation, we obtain the condition (saturated) of the first order as follows:

$$-Y_A \left[\frac{-2T n_A}{(T^2 + n_A)^2} + \bar{d} \frac{-2T n_{CA} + d_{\dot{C}A} T^2}{(n_{CA} - T d_{\dot{C}A} + T^2)^2}\right] - 1 = 0 \tag{32}$$

Similarly, the problem of institution B is stated as follows:

$$\max_\lambda \left[1 - \left(\sum_{i=1}^{\cdot} \alpha_B^i + (1 - \bar{d}) \sum_{j=1}^{\cdot} \alpha_{CB}^j\right)\right] Y_B - \lambda T^*$$
$$= \max_\lambda \left[1 - \left(\sum_{i=1}^{\cdot} \frac{e_B^i}{\sum e_B^i + \lambda T^*} + (1 - \bar{d}) \sum_{j=1}^{\cdot} \frac{e_{CB}^j}{\sum e_{CB}^j + \lambda T^*}\right)\right] Y_B - \lambda T^*.$$

Deriving this expression by $\lambda\ T^*$, we obtain the following condition:

$$-Y_B\left[\frac{-2\lambda T^* n_B}{\left((\lambda T^*)^2+n_B\right)^2}+(1-\bar{d})\frac{-2n_{CB}\lambda T^*+d_{\dot{C}B}(\lambda T^*)^2}{\left(n_{CB}-d_{\dot{C}B}\lambda T^*+(\lambda T^*)^2\right)^2}\right]-1=0 \qquad (33)$$

Using these two equations, we finally have the expression of $\bar{d}$ that is given as:

$$\bar{d}=\frac{Y_A\frac{-2Tn_A}{\left(T^2+n_A\right)^2}+Y_B\frac{2\lambda T^* n_B}{\left((\lambda T^*)^2+n_B\right)^2}-Y_B\frac{-2n_{CB}\lambda T^*+d_{\dot{C}B}(\lambda T^*)^2}{\left(n_{CB}-d_{\dot{C}B}\lambda T^*+(\lambda T^*)^2\right)^2}}{-Y_A\frac{-2Tn_{CA}+d_{\dot{C}A}T^2}{\left(n_{CA}-Td_{\dot{C}A}+T^2\right)^2}-Y_B\frac{-2n_{CB}\lambda T^*+d_{\dot{C}B}(\lambda T^*)^2}{\left(n_{CB}-d_{\dot{C}B}\lambda T^*+(\lambda T^*)^2\right)^2}}$$

Note that $\bar{d}$ is a function of the endowments T, $\lambda\ T^*$ of the institutions, the efforts of the players, and other parameters of the problem. It is the distance rendering the thief indifferent to operating in A or B. Let us call this equilibrium result $\bar{d}^*$.

$$\bar{d}^*=\frac{Y_A\frac{-2T^* n_A}{\left(T^{*2}+n_A\right)^2}+Y_B\frac{2\lambda^* T^* n_B}{\left((\lambda^* T^*)^2+n_B\right)^2}-Y_B\frac{-2n_{CB}\lambda^* T^*+d_{\dot{C}B}(\lambda^* T^*)^2}{\left(n_{CB}-d_{\dot{C}B}\lambda^* T^*+(\lambda^* T^*)^2\right)^2}}{-Y_A\frac{-2T^* n_{CA}+d_{\dot{C}A}T^{*2}}{\left(n_{CA}-T^* d_{\dot{C}A}+T^{*2}\right)^2}-Y_B\frac{-2n_{CB}\lambda^* T^*+d_{\dot{C}B}(\lambda^* T^*)^2}{\left(n_{CB}-d_{\dot{C}B}\lambda^* T^*+(\lambda^* T^*)^2\right)^2}} \qquad (34)$$

Note that $\bar{d}^*$ is a function of the endowments T^*, $\lambda^* T^*$ of the institutions at equilibrium, the efforts of the players, and other parameters of the problem.

4.2 *Equilibrium deterrence endowments*

The equilibrium endowments of the institutions, denoted as T^* and $\lambda^* T^*$, are derived from the following system of equations:

$$\begin{cases} -Y_A\left[\dfrac{-2Tn_A}{(T^2+n_A)^2}+\bar{d}*\dfrac{-2Tn_{CA}+d_{CA}T^2}{(n_{CA}-Td_{CA}+T^2)^2}\right]-1=0 \\ -Y_B\left[\dfrac{-2\lambda T*n_B}{((\lambda T*)^2+n_B)^2}+(1-\bar{d}*)\dfrac{-2n_{CB}\lambda T*+d_{CB}(\lambda T*)^2}{(n_{CB}-d_{CB}\lambda T*+(\lambda T*)^2)^2}\right]-1=0 \end{cases} \qquad (35)$$

We have two equations and two unknowns that are nothing more than the deterrent endowments of the two institutions. This system theoretically admits a solution. Several equilibrium situations can be examined from this system of equations. The first equation can help us to determine the value of T^* and the second equation the value of λ^* at equilibrium.

$$-Y_A\left[\frac{-2Tn_A}{\left(T^2+n_A\right)^2}+\bar{d}^*\frac{-2Tn_{CA}+d_{\dot{C}A}T^2}{\left(n_{CA}-Td_{\dot{C}A}+T^2\right)^2}\right]-1=0$$

We can write:

$$-Y_A\left[\frac{-2Tn_A(n_{CA}-Td_{\dot{C}A}+T^2)^2+\bar{d}^*(-2Tn_{CA}+d_{\dot{C}A}T^2)(T^2+n_A)^2}{(T^2+n_A)^2(n_{CA}-Td_{\dot{C}A}+T^2)^2}\right]-1=0$$

$$\begin{aligned}&-Y_A\left[-2Tn_A\left(n_{CA}-Td_{\dot{C}A}+T^2\right)^2+\bar{d}^*\left(-2Tn_{CA}+d_{\dot{C}A}T^2\right)\left(T^2+n_A\right)^2\right]\\&\quad-\left(T^2+n_A\right)^2\left(n_{CA}-Td_{\dot{C}A}+T^2\right)^2=0\end{aligned}$$

Finally, we have:

$$\begin{aligned}&-T^8+2d_{\dot{C}A}T^7+\left(-d_{\dot{C}A}{}^2-2n_{CA}-2n_A-Y_A\bar{d}^*d_{\dot{C}A}\right)T^6\\&\quad+T^5\left(2n_{CA}d_{\dot{C}A}+4n_Ad_{\dot{C}A}+2Y_A\bar{d}^*n_{CA}+2Y_An_A\right)\\&\quad+T^4\left(-n_{CA}{}^2-2n_Ad_{\dot{C}A}{}^2-n_A{}^2-2Y_A\bar{d}^*d_{\dot{C}A}n_A-4Y_An_Ad_{\dot{C}A}\right)\\&\quad+T^3\left(4n_An_{CA}d_{\dot{C}A}+2d_{\dot{C}A}n_A{}^2+4n_An_{CA}Y_A\bar{d}^*+2n_AY_A(2n_{CA}+d_{\dot{C}A}{}^2)\right)\\&\quad+T^2\left(-2n_An_{CA}{}^2-n_A{}^2d_{\dot{C}A}{}^2-2n_{CA}n_A{}^2-Y_A\bar{d}^*d_{\dot{C}A}n_A{}^2-4Y_An_An_{CA}d_{\dot{C}A}\right)\\&\quad+T\left(2n_{CA}d_{\dot{C}A}n_A{}^2+2Y_A\bar{d}^*n_{CA}n_A{}^2+2Y_An_An_{CA}{}^2\right)-n_A{}^2n_{CA}{}^2=0\end{aligned} \tag{36}$$

The value of institution A's balancing endowment at equilibrium is given by the intersection of the function curves

$$\begin{aligned}&f(T)=\\&-T^8+2d_{\dot{C}A}T^7+\left(-d_{\dot{C}A}{}^2-2n_{CA}-2n_A-Y_A\bar{d}^*d_{\dot{C}A}\right)T^6\\&+T^5\left(2n_{CA}d_{\dot{C}A}+4n_Ad_{\dot{C}A}+2Y_A\bar{d}^*n_{CA}+2Y_An_A\right)\\&+T^4\left(-n_{CA}{}^2-2n_Ad_{\dot{C}A}{}^2-n_A{}^2-2Y_A\bar{d}^*d_{\dot{C}A}n_A-4Y_An_Ad_{\dot{C}A}\right)\\&+T^3\left(4n_An_{CA}d_{\dot{C}A}+2d_{\dot{C}A}n_A{}^2+4n_An_{CA}Y_A\bar{d}^*+2n_AY_A\left(2n_{CA}+d_{\dot{C}A}{}^2\right)\right)\\&+T^2\left(-2n_An_{CA}{}^2-n_A{}^2d_{\dot{C}A}{}^2-2n_{CA}n_A{}^2-Y_A\bar{d}^*d_{\dot{C}A}n_A{}^2-4Y_An_An_{CA}d_{\dot{C}A}\right)\end{aligned}$$

and $g(T)=T\left(2n_{CA}d_{\dot{C}A}n_A{}^2+2Y_A\bar{d}^*n_{CA}n_A{}^2+2Y_An_An_{CA}{}^2\right)-n_A{}^2n_{CA}{}^2$.

The derivative of $f(T)$ is given by

$$
\begin{aligned}
&f'(T) = \\
&-\Big[8T^7 + 6\left(d_{\dot{C}A}{}^2 + 2n_{CA} + 2n_A + 2Y_A\bar{d}^* d_{\dot{C}A}\right)T^5 \\
&+ 4\left(n_{CA}{}^2 + 2n_A d_{\dot{C}A}{}^2 + n_A{}^2 + 2Y_A\bar{d}^* d_{\dot{C}A} n_A + 4Y_A n_A d_{\dot{C}A}\right)T^3 \\
&+ 2\left(2n_A n_{CA}{}^2 + n_A{}^2 d_{\dot{C}A}{}^2 + 2n_{CA} n_A{}^2 + Y_A\bar{d}^* d_{\dot{C}A} n_A{}^2 + 4Y_A n_A n_{CA} d_{\dot{C}A}\right)T\Big] \\
&+ \Big[14 d_{\dot{C}A} T^6 + 5\left(2n_{CA} d_{\dot{C}A} + 4n_A d_{\dot{C}A} + 2Y_A\bar{d}^* n_{CA} + 2Y_A n_A\right)T^4 \\
&+ 3(4n_A n_{CA} d_{\dot{C}A} + 2d_{\dot{C}A} n_A{}^2 + 4n_A n_{CA} Y_A\bar{d}^* + 2n_A Y_A\left(2n_{CA} + d_{\dot{C}A}{}^2\right))T^2\Big].
\end{aligned}
$$

The first term is negative, while the second term is positive. However, in terms of absolute value, the first term is more important than the second. The derivative of $f(T)$ vanishes at $T = 0$. This derivative is negative for any value of $T > 0$. Conversely, the derivative of $g(T)$ is positive. The opposite signs of these derivatives enable us to say that the two curves grow at a single point T, which is the equilibrium point in terms of endowment for institution A.

The second equation is

$$
-Y_B\left[\frac{-2\lambda T^* n_B}{\left((\lambda T^*)^2 + n_B\right)^2} + \left(1 - \bar{d}^*\right)\frac{-2n_{CB}\lambda T^* + d_{\dot{C}B}(\lambda T^*)^2}{\left(n_{CB} - d_{\dot{C}B}\lambda T^* + (\lambda T^*)^2\right)^2}\right] - 1 = 0
$$

We can write:

$$
-Y_B\left[\frac{-2\lambda T^* n_B(n_{CB} - d_{\dot{C}B}\lambda T^* + (\lambda T^*)^2)^2 + (1-\bar{d}^*)(-2n_{CB}\lambda T^* + d_{\dot{C}B}(\lambda T^*)^2)((\lambda T^*)^2 + n_B)^2}{((\lambda T^*)^2 + n_B)^2(n_{CB} - d_{\dot{C}B}\lambda T^* + (\lambda T^*)^2)^2}\right] - 1 = 0
$$

$$
\begin{aligned}
&-Y_B\Big[-2\lambda T^* n_B(n_{CB} - d_{\dot{C}B}\lambda T^* + (\lambda T^*)^2)^2 \\
&\quad + (1-\bar{d}^*)(-2n_{CB}\lambda T^* + d_{\dot{C}B}(\lambda T^*)^2)((\lambda T^*)^2 + n_B)^2\Big] \\
&\quad - \left[(\lambda T^*)^2 + n_B\right]^2\left[(n_{CB} - d_{\dot{C}B}\lambda T^* + (\lambda T^*)^2\right]^2 = 0
\end{aligned}
$$

Finally, we have:

$$\begin{aligned}
&-T^{*8}\lambda^8+2d_{\dot{C}B}T^{*7}\lambda^7+\left(-d_{\dot{C}B}{}^2-2n_{CB}-2n_B-Y_B(1-\bar{d}^*)d_{\dot{C}B}\right)T^{*6}\lambda^6\\
&\quad+T^{*5}\left(2n_{CB}d_{\dot{C}B}+4n_Bd_{\dot{C}B}+2Y_B(1-\bar{d}^*)n_{CB}+2Y_Bn_B\right)\lambda^5\\
&\quad+T^{*4}\left(-n_{CB}{}^2-2n_Bd_{\dot{C}B}{}^2-n_B{}^2-2Y_B(1-\bar{d}^*)d_{\dot{C}B}n_B-4Y_Bn_Bd_{\dot{C}B}\right)\lambda^4\\
&\quad+T^{*3}\Big(4n_Bn_{CB}d_{\dot{C}B}+2d_{\dot{C}B}n_B{}^2+4n_Bn_{CB}Y_B(1-\bar{d}^*)\\
&\qquad\qquad+2n_BY_B\left(2n_{CB}+d_{\dot{C}B}{}^2\right)\Big)\lambda^3\\
&\quad+T^{*2}\Big(-2n_Bn_{CB}{}^2-n_B{}^2d_{\dot{C}B}{}^2-2n_{CB}n_B{}^2-Y_B(1-\bar{d}^*)d_{\dot{C}B}n_B{}^2\\
&\qquad\qquad-4Y_Bn_Bn_{CB}d_{\dot{C}B}\Big)\lambda^2\\
&\quad+T^*\left(2n_{CB}d_{\dot{C}B}n_B{}^2+2Y_B(1-\bar{d}^*)n_{CB}n_B{}^2+2Y_Bn_Bn_{CB}{}^2\right)\lambda-n_B{}^2n_{CB}{}^2=0.
\end{aligned}\tag{37}$$

The value of λ at equilibrium is given by the intersection of the function curves

$$\begin{aligned}
&f(\lambda)=\\
&-T^{*8}\lambda^8+2d_{\dot{C}B}T^{*7}\lambda^7+\left(-d_{\dot{C}B}{}^2-2n_{CB}-2n_B-Y_B(1-\bar{d}^*)d_{\dot{C}B}\right)T^{*6}\lambda^6\\
&+T^{*5}\left(2n_{CB}d_{\dot{C}B}+4n_Bd_{\dot{C}B}+2Y_B(1-\bar{d}^*)n_{CB}+2Y_Bn_B\right)\lambda^5\\
&+T^{*4}\left(-n_{CB}{}^2-2n_Bd_{\dot{C}B}{}^2-n_B{}^2-2Y_B(1-\bar{d}^*)d_{\dot{C}B}n_B-4Y_Bn_Bd_{\dot{C}B}\right)\lambda^4\\
&+T^{*3}\left(4n_Bn_{CB}d_{\dot{C}B}+2d_{\dot{C}B}n_B{}^2+4n_Bn_{CB}Y_B(1-\bar{d}^*)+2n_BY_B\left(2n_{CB}+d_{\dot{C}B}{}^2\right)\right)\lambda^3\\
&+T^{*2}\Big(-2n_Bn_{CB}{}^2-n_B{}^2d_{\dot{C}B}{}^2-2n_{CB}n_B{}^2-Y_B(1-\bar{d}^*)d_{\dot{C}B}n_B{}^2\\
&\qquad-4Y_Bn_Bn_{CB}d_{\dot{C}B}\Big)\lambda^2
\end{aligned}$$

and

$$g(\lambda)=T^*\left(2n_{CB}d_{\dot{C}B}n_B{}^2+2Y_B(1-\bar{d}^*)n_{CB}n_B{}^2+2Y_Bn_Bn_{CB}{}^2\right)\lambda-n_B{}^2n_{CB}{}^2.$$

The derivative of $f(\lambda)$ is:

$$
\begin{aligned}
&f'(\lambda) = \\
&\Big[8T^{*8}\lambda^7 + 6\left(d_{CA}^{\cdot}{}^2 + 2n_{CA} + 2n_A + 2Y_A(1-\bar{d}^*)d_{CA}^{\cdot}\right)T^{*6}\lambda^5 \\
&+ 4\left(n_{CA}{}^2 + 2n_A d_{CA}^{\cdot}{}^2 + n_A{}^2 + 2Y_A(1-\bar{d}^*)d_{CA}^{\cdot} n_A + 4Y_A n_A d_{CA}^{\cdot}\right)T^{*4}\lambda^3 \\
&+ 2\big(2n_A n_{CA}{}^2 + n_A{}^2 d_{CA}^{\cdot}{}^2 + 2n_{CA} n_A{}^2 + Y_A(1-\bar{d}^*)d_{CA}^{\cdot} n_A{}^2 \\
&\qquad + 4Y_A n_A n_{CA} d_{CA}^{\cdot}\big)T^{*2}\lambda\Big] \\
&+ \Big[14 d_{CA}^{\cdot} T^{*7}\lambda^6 + 5\left(2n_{CA} d_{CA}^{\cdot} + 4n_A d_{CA}^{\cdot} + 2Y_A(1-\bar{d}^*)n_{CA} + 2Y_A n_A\right)T^{*5}\lambda^4 \\
&+ 3\big(4n_A n_{CA} d_{CA}^{\cdot} + 2d_{CA}^{\cdot} n_A{}^2 + 4n_A n_{CA} Y_A(1-\bar{d}^*) \\
&\qquad + 2n_A Y_A\left(2n_{CA} + d_{CA}^{\cdot}{}^2\right)\big)T^{*3}\lambda^2\Big].
\end{aligned}
$$

The first term is negative, while the second term is positive. However, in terms of absolute value, the first term is more important than the second. The derivative of $f(\lambda)$ vanishes at $\lambda = 0$. This derivative is negative for any value of $\lambda > 0$. In contrast, the derivative of $g(\lambda)$ is positive. The opposite signs of these derivatives enable us to say that the two curves grow at a single point λ, which is the equilibrium point, and in terms of endowment for institution B, we have $\lambda^* T^*$.

The solutions are given by the following figures (figures 1 and 2 in annex).

The solutions are of the form $T^* = h(Y_A)$ and $\lambda^* = h(Y_B,\ T^*)$.

4.3 *Equilibrium values of thieves' efforts*

Let us say that e_A^*, e_{CA}^*, e_A^*, e_{CB}^* are the equilibrium values of thieves' efforts; then, we have:

$$
\begin{aligned}
e_A^* &= \frac{1}{T^*} = \frac{1}{h(Y_A)}; \\
e_B^* &= \frac{1}{\lambda^* T^*} = \frac{1}{h(Y_B)}; \\
e_{CA}^* &= \frac{1}{T^*} - d_{CA} = \frac{1}{h(Y_A)} - d_{CA}; \\
e_{CB}^* &= \frac{1}{\lambda^* T^*} - d_{CB} = \frac{1}{h(Y_B)} - d_{CB}.
\end{aligned}
$$

In the following sections, we attempt to analyse the different types of optimum.

4.4 *Efficiency*

$$\max_{T,\ \lambda T^*}\left[1-\left(\sum_{i=1}^{\cdot}\alpha_B^i+(1-\bar{d})\sum_{j=1}^{\cdot}p_{CB}^j\right)\right]Y_B + \left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^i+\bar{d}\sum_{j=1}^{\cdot}p_{CA}^j\right)\right]Y_A - T-\lambda T^* \tag{38}$$

$$\text{s/c } \bar{d}_1+\bar{d}_3 e_{A1}^i \leq 0 \text{ and } \bar{d}_2+\bar{d}_4 e_{B1}^i \geq 0 \tag{39}$$

As a reminder, these constraints make it possible to avoid the perverse effect of the reduction in the effort of thieves in production spaces on the stolen production following the increase in institutional endowments. This reduction in the efforts e_A^i and e_B^i induces the thieves in space C to operate in spaces A and B. Considering that these constraints are saturated, $\bar{d}_1=-\bar{d}_3 e_{A1}^i$ and $\bar{d}_2=-\bar{d}_4 e_{B1}^i$, and by integrating them under the first-order conditions, we have:

$$-\left[\sum_{i=1}^{\cdot}\alpha_{A2}^i e_{A1}^i+\bar{d}(\sum_{j=1}^{\cdot}p_{CA2}^j e_{CA1}^j)\right]Y_A-\left[\sum_{i=1}^{\cdot}\alpha_{A1}^i+\bar{d}\left(\sum_{j=1}^{\cdot}p_{CA1}^j\right)\right]Y_A-1=0 \tag{40}$$

From equation (34), replacing $\bar{d}_3 e_{A1}^i$ by $-\bar{d}_1$, we have

$$-\left[\sum_{i=1}^{\cdot}\alpha_{A2}^i e_{A1}^i+\bar{d}(\sum_{j=1}^{\cdot}p_{CA2}^j e_{CA1}^j)\right]Y_A-\left[\sum_{i=1}^{\cdot}\alpha_{A1}^i+\bar{d}\left(\sum_{j=1}^{\cdot}p_{CA1}^j\right)\right]Y_A-1=0 \tag{41}$$

which results in:

$$-\left[\sum_{i=1}^{\cdot}\alpha_{A2}^i e_{A1}^i+\bar{d}(\sum_{j=1}^{\cdot}p_{CA2}^j e_{CA1}^j)\right]Y_A-\left[\sum_{i=1}^{\cdot}\alpha_{A1}^i+\bar{d}\left(\sum_{j=1}^{\cdot}p_{CA1}^j\right)\right]Y_A=1. \tag{42}$$

The optimum can take different forms, which are analysed in the following sections.

4.5 *Interior and Symmetrical Optimum*

For an interior and symmetrical optimum, we have $T=\lambda T=T^{**}>0$ and $\bar{d}^*=1-\bar{d}^*=\frac{1}{2}$. As $\bar{d}=\bar{d}^*(T^{**},e^*)$, it can be deduced that $e^*=e_B^*=e_A^*$ and $\lambda^*=1$. By integrating these data into the first-order conditions and considering n to be the number of thieves in A and m to be the number in C operating in A, we can finally write that:

$$(-n_A \alpha_{A2}^{**} e_{A1}^{*} - \frac{1}{2} n_{CA} p_{CA2}^{**} e_{CA1}^{*} - n_A \alpha_{A1}^{**} - \frac{1}{2} n_{CA} p_{CA1}^{**}) Y_A = 1 \tag{43}$$

which is equal to

$$\left[- n_A \left(\alpha_{A2}^{**} e_{A1}^{*} + \alpha_{A1}^{**} \right) \right] Y_A + \frac{1}{2} \left[- n_C A (p_C A2^{**} e_C A1^{*} + p_C A1^{**} \right] Y_A = 1 \tag{44}$$

Proposal 3

The *direct* total marginal reduction in the proportion of production stolen in A by the thieves in A and C, which results from the increase in the deterrent endowment of institution A, plus the *indirect* total marginal reduction in the proportion of production stolen in A by the thieves in A and C, which results from the reduction in their effort through the increase in institution A's deterrence grant, is equal to the marginal reduction in other A expenditures required to finance the increase in deterrence in space A. For proof, see equation (44).

This result, which differs from the results in the literature, can be explained by the difference introduced between the thieves in spaces A and C, thus revealing the indirect effects. It indicates that the institutional endowment makes it possible to reduce, on the one hand, the proportion of the thieves and, on the other hand, the effort of the thieves who will persevere in the activity of robbery. The symmetric optimum is given by

$$\begin{aligned} &\left[1 - \left(\sum_{i=1}^{n_A} \alpha_A^{i**} + \frac{1}{2} \sum_{j=1}^{n_{CA}} p_{CA}^{j**} \right) \right] Y_A - T^{**} \\ &= \left[1 - \left(\sum_{i=1}^{n_B} \alpha_B^{i**} + \frac{1}{2} \sum_{j=1}^{n_{CB}} p_{CB}^{j**} \right) \right] Y_B - T^{**} \end{aligned} \tag{45}$$

Three cases can arise: $Y_A = Y_B$; $Y_A < Y_B$ and $Y_A > Y_B$.

When $Y_A = Y_B$, we obtain

$$\begin{aligned} &\left[1 - \left(\sum_{i=1}^{n_A} \alpha_A^{i**} + \frac{1}{2} \sum_{j=1}^{n_{CA}} p_{CA}^{j**} \right) \right] Y - T^{**} \\ &= \left[1 - \left(\sum_{i=1}^{n_B} \alpha_B^{i**} + \frac{1}{2} \sum_{j=1}^{n_{CB}} p_{CB}^{j**} \right) \right] Y - T^{**} \end{aligned} \tag{46}$$

In this case, the symmetric equilibrium is such that the gain of a thief in C is identical whether he or she operates in A or B. Similarly, the gain of thieves positioned in A and B is identical. We finally have a *stable* equilibrium.

When $Y_A < Y_B$ and considering the example that $2Y_A = Y_B = Y$, we obtain:

$$\begin{aligned}&\frac{1}{2}\left[1-\left(\sum_{i=1}^{n_A}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{n_{CA}}p_{CA}^{j**}\right)\right]Y-T^{**}\\&=\left[1-\left(\sum_{i=1}^{n_B}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{n_{CB}}p_{CB}^{j**}\right)\right]Y-T^{**}\end{aligned} \tag{47}$$

In this case, the symmetric equilibrium is such that the gain of the thief positioned in B is greater than the gain of the thief positioned in A. Similarly, the gain of the thief in C operating in B is greater than the gain of the thief in C operating in A. Therefore, we have an *unstable* equilibrium.

When $Y_A > Y_B$ and considering the example of $Y_A = 2Y_B = Y$, we obtain:

$$\begin{aligned}&\left[1-\left(\sum_{i=1}^{n_A}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{n_{CA}}p_{CA}^{j**}\right)\right]Y-T^{**}\\&=\frac{1}{2}\left[1-\left(\sum_{i=1}^{n_B}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{n_{CB}}p_{CB}^{j**}\right)\right]Y-T^{**}\end{aligned} \tag{48}$$

In this case, the symmetric equilibrium is such that the gain of the thief positioned in B is less than the gain of the thief positioned in A. Similarly, the gain of the thief in C operating in B is less than the gain of the thief in C operating in A, thus creating an *unstable* equilibrium.

It is concluded that the only *stable symmetric* equilibrium is that the production levels in the two institutions are identical.

4.6 *Asymmetric Optimum*

It is possible that the optimum is such that all thieves operate in one institution. Let us consider the case in which the thieves all operate in A. We note $m = n_{CA} + n_{CB}$. The maximization program is written as:

$$\max_T \left[1-\left(\sum_{i=1}^{n_A}\alpha_A^i+\bar{d}\sum_{j=1}^{m}\alpha_{CA}^j\right)\right]Y_A - Ts/c\bar{d} = 1 \tag{49}$$

Considering this constraint, we finally have:

$$\max_T \left[1-\left(\sum_{i=1}^{n_A}\alpha_A^i+\sum_{j=1}^{m}\alpha_{CA}^j\right)\right]Y_A - T \tag{50}$$

The first-order conditions are:

$$\left[-\left(\sum_{i=1}^{n_A}\alpha_{A1}^{i}+\sum_{j=1}^{m}\alpha_{CA1}^{j}\right)-\left(\sum_{i=1}^{n_A}\alpha_{A2}^{i}e_{A1}^{i}+\sum_{j=1}^{m}\alpha_{CA2}^{j}e_{CA1}^{i}\right)\right]Y_A-1=0 \quad (51)$$

Let us consider T^* the solution to this condition. Given T^*, the choice of institution B is such that no thief wants to operate in B, so $\bar{d}\,(T^*,\ \lambda^*T^*,\ e_A^*,\ 0)=1$.

Proposal 4

If

$$\left[1-\left(\sum_{i=1}^{n_B}\alpha_B^i(\lambda^*T^*,0)\right)\right]Y_B-\lambda^*T^*$$
$$=\left[1-\left(\sum_{i=1}^{n_A}\alpha_A^i\,(T^*,e_A^*)+\sum_{j=1}^{m}p_{CA}^{j}(T^*,e_{CA}^*,1)\right)\right]Y_A-T^*$$

and in other words,

$$[Y_B-\lambda^*T^*]=\left[1-\left(\sum_{i=1}^{n}\alpha_A^i\,(T^*,e_A^*)+\sum_{j=1}^{m}p_{CA}^{j}(T^*,e_{CA}^*,1\right)\right]Y_A-T^*,$$

the choice deterrence levels in the asymmetric equilibrium without coordination between institutions correspond to those of the asymmetric optimum.

Proof of proposal 4: In asymmetric equilibrium, all of the thieves in C operate in A, and no thief in B exerts a strictly positive effort. Institution B must provide a level of deterrence λ^*T^* that urges thieves in B to remain inactive. The net gain of institution B is therefore $Y_B-\lambda^*T^*$. In institution A, losses are placed at three levels: loss due to thieves in A, loss due to all thieves in C, and deterrent endowment. Hence, the net gain is assessed as follows:

$$\left[1-\left(\sum_{i=1}^{n_A}\alpha_A^i\,(T^*,e_A^*)+\sum_{j=1}^{m}p_{CA}^{j}(T^*,e_{CA}^*,1)\right)\right]Y_A-T^* \quad (52)$$

The asymmetric equilibrium assumes that the two gains are identical, leading to proposition 4. There are three cases: the case in which $Y_B=Y_A$, the case in which $Y_B>Y_A$ and the case in which $Y_B<Y_A$. We considering these cases.

The case in which $Y_B = Y_A = Y$

We finally have

$$[Y - \lambda^* T^*] = Y - \left[\left(\left(\sum_{i=1}^{n_A} \alpha_A^i (T^*, e_A^*) + \sum_{j=1}^{m} p_{CA}^j (T^*, e_{CA}^*, 1\right)\right) Y + T^*\right] \quad (53)$$

Proposal 5

If the level of production is the same in both institutions, when institution A accepts that all of the thieves in C operate in its territorial space, institution B chooses only a minimum level of deterrence to urge only the thieves in B not to exert any strictly positive effort. However, this effort is found to be very important compared with the deterrence effort of institution A. The endowment $\lambda^* T^*$ in B that enables no thief in B to provide a positive effort is equal to the sum of the loss in A and the institutional endowment T^* (see equation (53)). This negative externality has not been specified in classical analyses because of inadequacies in modelling. Indeed, that all thieves in C operate in A improves the performance of thieves in B. A higher endowment is therefore necessary to compel them not to exert a strictly positive effort. This result raises the issue noted in the conclusion of Marceau and Mongrain (2002) that the externality disappears in an asymmetric equilibrium. The modification of the environment of the model proposed by Marceau and Mongrain (2002) leads to different and more realistic results. This result shows that if, there were another CBG in the model of Zenou (2003) in the residential space, all of the thieves would not go to operate in the initial CBD. The existence of a second CBD in which there is a production could lead to new findings in terms of crime committed by thieves. Therefore, thieves must choose between the first or second CBD, which was not the case in Zenou (2003). Thereby, the choice of theft depends on: the distance that the theft must cover to reach the production; and his or her effort, the efforts of other thieves and the distance rendering the thief indifferent to the idea of operating in initial CBD or second CBD.

The case in which $Y_B > Y_A$

We consider as an illustration the case in which $Y_B = 2Y_A = Y$. We can write:

$$[Y - \lambda^* T^*] = \frac{1}{2} Y - \frac{1}{2}\left[\left(\sum_{i=1}^{n_A} \alpha_A^i (T^*, e_A^*) + \sum_{j=1}^{m} p_{CA}^j (T^*, e_{CA}^*, 1)\right) Y + 2T^*\right] \quad (54)$$

such that

$$\begin{aligned}&\left[Y-\lambda^{*}T^{*}\right]=\\&\frac{1}{2}Y-\frac{1}{2}\left[\left(\left(\sum_{i=1}^{n_A}\alpha_A^i\left(T^{*},e_A^{*}\right)+\sum_{j=1}^{m}p_{CA}^{j}(T^{*},e_{CA}^{*},1\right)\right)Y+T^{*}\right]-\frac{1}{2}T^{*}\end{aligned} \tag{55}$$

The double of what would remain in B is equal to what should have remained in A if the level of production we that of B minus the deterrent endowment. Equation (55) leads to proposal 6.

Proposal 6

When production in B is greater than production in A, in an asymmetric equilibrium where all thieves in C operate in A and no thief in B provides a strictly positive effort, the remaining market output in B is lower than the remaining non-market output in A. In other words, the welfare under institution A is greater than the welfare under institution B. Intuitively, the institutional endowment needed to urge thieves in B not to exert a strictly positive effort becomes very high, and everything suggests that the action of thieves in C operating in B reduces the efficiency of the thieves' efforts in B. A confrontation between thieves from different areas could be a means for the institution to provide a lesser deterrent in an asymmetric equilibrium while maintaining a high level of well-being, compared to the competing institution.

The case in which $Y_B < Y_A$, for example, $2Y_B = Y_A = Y$

We have

$$\left[\frac{1}{2}Y-\lambda^{*}T^{*}\right]=Y-\left[\left(\left(\sum_{i=1}^{n_A}\alpha_A^i\left(T^{*},e_A^{*}\right)+\sum_{j=1}^{m}p_{CA}^{j}(T^{*},e_{CA}^{*},1\right)\right)Y+T^{*}\right] \tag{56}$$

We finally obtain

$$\begin{aligned}&\frac{1}{2}\left[Y-\lambda^{*}T^{*}\right]=\\&Y-\left[\left(\left(\sum_{i=1}^{n_A}\alpha_A^i\left(T^{*},e_A^{*}\right)+\sum_{j=1}^{m}p_{CA}^{j}(T^{*},e_{CA}^{*},1\right)\right)Y+T^{*}\right]+\frac{1}{2}\lambda^{*}T^{*}\end{aligned} \tag{57}$$

Proposal 7

When production in B is less important than production in A, in an asymmetric equilibrium in which all thieves in C operate in A, and no thief in B exerts a strictly positive effort, the market output remaining in B is more important than the non-market output remaining in A. In other words, welfare in institution B is greater than welfare in institution A. Intuitively, the institutional endowment needed to compel thieves in B not to exert a strictly positive effort decreases when production in B is less than output in A.

Equation (57) leads to proposition 7. In conclusion, in an asymmetric equilibrium in which all thieves in C operate under institution A and in which no thief provides a strictly positive effort in B, institution B has an interest in having a lower level of production compared to A if it wishes to have a well-being superior to that of institution A. This result seems paradoxical. However, it indicates that the best asymmetric equilibrium for an institution that does not face robbery issues is to have lower production than the institution facing theft. The difference in factorial and technological endowment in favour of a locality might be preferable in a situation of asymmetric equilibrium between two localities. This result validates the empirical results of Draca and Machin (2015).

4.7 *Global Optimum and Intervention*

Let us consider T^{**} to be the level of deterrence of the symmetric optimum and $(T^*,\ \lambda^* T^*)$ the deterrence levels at the asymmetric optimum in which no criminal operates in A or B. The sum of the net gains of the two institutions in the asymmetric optimum is given by:

$$\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y_A-T^*+Y_B-\lambda^*T^* \tag{58}$$

Similarly, the sum of the net gains in the symmetric optimum is given by:

$$\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y_A+\left[1-\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y_B-2T^* \tag{59}$$

For the asymmetric optimum to be an overall optimum, it is sufficient that the sum of the net gains of the two institutions with asymmetric optimum is greater than the sum of the net gains in symmetric optimum. Let us consider the following condition:

$$\begin{aligned}&\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y_A+Y_B-\left[\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y_A\right.\\&\left.+\left[1-\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y_B\right]\geq[T^*+\lambda^*T^*]-2T^{**}.\end{aligned} \tag{60}$$

There are three cases: the case in which $Y_B = Y_A$, the case in which $Y_B > Y_A$ and the case in which $Y_B < Y_A$.

The case in which $Y_B = Y_A = Y$

In the integration of this relation into equation (60), we have:

$$\begin{aligned}&\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y+Y-\left[\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y\right.\\&\left.+\left[1-\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y\right]\geq[T^*+\lambda^*T^*]-2T^{**}\end{aligned}\tag{61}$$

We finally have:

$$\begin{aligned}&\left[\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right]Y+\left[\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right]Y\\&-\left[\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right]Y\geq[T^*+\lambda^*T^*]-2T^*\end{aligned}\tag{62}$$

This result differs from the result of Marceau and Mongrain (2002). The new condition is that the asymmetric optimum is an overall optimum if the difference between the total proportion of production stolen under institution A at the symmetric optimum and the total proportion of production stolen under institution A at the asymmetric optimum is greater than the difference between the deterrence costs in both optima.

The case in which $Y_B > Y_A$, with $Y_B = 2Y_A = Y$

By integrating the relation into equation (60), we have:

$$\begin{aligned}&\frac{1}{2}\left[\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y+\left[\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y\\&-\frac{1}{2}\left[\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y\geq[T_A^*+\lambda^*T^*]-2T^{**}\end{aligned}\tag{63}$$

In conclusion, the asymmetric optimum remains an overall optimum, although market production is lower than non-market output.

The case in which $Y_B < Y_A$, with $2Y_B = Y_A = Y$

By integrating the relation into equation (60), we have:

$$\begin{aligned}&\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y-\left[\left[1-\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y\right.\\&\left.+\frac{1}{2}\left[1-\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y\right]\geq[T^*+\lambda^*T^*]-2T^{**}\end{aligned}\tag{64}$$

Finally, we have:

$$\begin{aligned}&\left[\left(\sum_{i=1}^{\cdot}\alpha_A^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CA}^{j**}\right)\right]Y+\frac{1}{2}\left[\left(\sum_{i=1}^{\cdot}\alpha_B^{i**}+\frac{1}{2}\sum_{j=1}^{\cdot}p_{CB}^{j**}\right)\right]Y\\&-\left[\left(\sum_{i=1}^{\cdot}\alpha_A^{i*}+\sum_{j=1}^{\cdot}p_{CA}^{j*}\right)\right]Y\geq[T^*+\lambda^*T^*]-2T^{**}\end{aligned}\tag{65}$$

In conclusion, the asymmetric optimum remains an overall optimum even in the case in which market production is higher than non-market output.

CONCLUSION

In this investigation, we use the Tullock contest function for n-players to identify thieves' efforts and institutions' endowments at equilibrium. At the end of this investigation, we note that symmetric equilibrium becomes unstable when the levels of production under the institutions are different. However, asymmetric equilibrium remains optimal, even in a situation of differences in production between institutions. The proportion of thieves in localities of production is of little importance in the choice of relocation of the thief to the centre. Conversely, the effort provided by each thief who is there is decisive in this choice. This result improves the social interactions view (Zenou, 2003), which stipulates that an individual is more likely to commit a crime if his or her peers commit it than if they do not commit any. In the case addressing theft, we should consider the competitive spirit that exists among thieves. Moreover, the article shows that, unlike Marceau and Mongrain (2002), in an asymmetric equilibrium situation, the externality does not disappear. A confrontation between thieves from different areas can be a way for the institution to provide less deterrence in an asymmetric balance while guaranteeing a higher level of consumption than that under the opposing institution. Finally, an increase in the deterrent endowment has a negative impact on the proportion of production stolen. This article argues for deterrence policies.

APPENDIX

Lemma 1: Proof

$$\begin{aligned}&\left[\alpha_{CA}\left(T,e^{j}_{CA},\bar{d}\right)\right]Y_A-\left(\bar{d}+e^{j}_{CA}\right)t\\&=\left[\alpha_{CB}\left(\lambda T,e^{j}_{CB},1-\bar{d}\right)\right]Y_B-\left(1-\bar{d}+e^{j}_{CB}\right)t\end{aligned}\tag{A1}$$

$$\begin{aligned}&\left[\alpha_{CA}\left(T,e^{j}_{CA},\bar{d}\right)\right]Y_A-\left(\bar{d}+e^{j}_{CA}\right)-\left[\alpha_{CB}\left(\lambda T,e^{j}_{CB},1-\bar{d}\right)\right]Y_B\\&-\left(1-\bar{d}+e^{j}_{CB}\right)t=0\end{aligned}\tag{A2}$$

We differentiate this equation with respect to T and considering that $T_B=\lambda T$ remain constant.

$$\begin{aligned}&Y_A\left(\frac{\partial\alpha_{CA}(.)}{\partial T}+\frac{\partial\alpha_{CA}(.)}{\partial e_{CA}}\frac{\partial e_{CA}}{\partial T}+\frac{\partial\alpha_{CA}(.)}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)-\left(\frac{\partial\bar{d}}{\partial T}+\frac{\partial\bar{d}}{\partial e_A}\frac{\partial e_A}{\partial T}\right)t\\&-\left(\frac{\partial e_{CA}}{\partial T}+\frac{\partial e_{CA}}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)t-\left(\frac{\partial\alpha_{CB}(.)}{\partial e_{CB}}\frac{\partial e_{CB}}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)Y_B+\left(\frac{\partial\alpha_{CB}}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)Y_B\\&+\left(-\frac{\partial\bar{d}}{\partial T}-\frac{\partial\bar{d}}{\partial e_A}\frac{\partial e_A}{\partial T}+\frac{\partial e_{CB}}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)t=0\end{aligned}\tag{A3}$$

Considering that the efforts remain constant, we have

$$\begin{aligned}&Y_A\left(\frac{\partial\alpha_{CA}(.)}{\partial T}+\frac{\partial\alpha_{CA}(.)}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)-\left(\frac{\partial\bar{d}}{\partial T}\right)t+\left(\frac{\partial\alpha_{CB}(.)}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)Y_B\\&+\left(-\frac{\partial\bar{d}}{\partial T}\right)t=0\end{aligned}\tag{A4}$$

$$\begin{aligned}&Y_A\left(\frac{\partial\alpha_{CA}(.)}{\partial T}+\frac{\partial\alpha_{CA}(.)}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)-\left(\frac{\partial\bar{d}}{\partial T}\right)t+\left(\frac{\partial\alpha_{CB}(.)}{\partial\bar{d}}\frac{\partial\bar{d}}{\partial T}\right)Y_B\\&+\left(-\frac{\partial\bar{d}}{\partial T}\right)t=0\end{aligned}\tag{A5}$$

$$\frac{\partial\bar{d}}{\partial T}\left(\frac{\partial\alpha_{CA}(.)}{\partial\bar{d}}Y_A+\frac{\partial\alpha_{CB}(.)}{\partial\bar{d}}Y_B-2t\right)=-\frac{\partial\alpha_{CA}(.)}{\partial T}Y_A\tag{A6}$$

$$\frac{\partial\bar{d}}{\partial T}=-\frac{\frac{\partial\alpha_{CA}(.)}{\partial T}Y_A}{\frac{\partial\alpha_{CA}(.)}{\partial\bar{d}}Y_A+\frac{\partial\alpha_{CB}(.)}{\partial\bar{d}}Y_B-2t}<0\tag{A7}$$

Hence, Lemma 1: Everything else being equal, $\bar{d}$ is decreasing in T.

We differentiate this equation with respect to λ or λT with T constant.

$$\begin{aligned} &Y_A\left(\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)-\left(\frac{\partial \bar{d}}{T\partial\lambda}+\frac{\partial \bar{d}}{\partial e_B}\frac{\partial e_B}{T\partial\lambda}\right)t-\left(\frac{\partial e_{CA}}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)t \\ &-\left(\frac{\partial \alpha_{CB}(.)}{\partial e_{CB}}\frac{\partial e_{CB}}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}+\frac{\partial \alpha_{CB}(.)}{T\partial\lambda}\right)Y_B+\left(\frac{\partial \alpha_{CB}}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)Y_B \\ &+\left(-\frac{\partial \bar{d}}{T\partial\lambda}-\frac{\partial \bar{d}}{\partial e_B}\frac{\partial e_B}{T\partial\lambda}+\frac{\partial e_{CB}}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)t=0 \end{aligned} \tag{A8}$$

Considering that the efforts remain constant, we have

$$\begin{aligned} &Y_A\left(\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)-\left(\frac{\partial \bar{d}}{T\partial\lambda}\right)t-\left(\frac{\partial \alpha_{CB}(.)}{T\partial\lambda}-\frac{\partial \alpha_{CB}}{\partial \bar{d}}\frac{\partial \bar{d}}{T\partial\lambda}\right)Y_B \\ &+\left(-\frac{\partial \bar{d}}{T\partial\lambda}\right)t=0 \end{aligned} \tag{A9}$$

$$\frac{\partial \bar{d}}{T\partial\lambda}\left(\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}Y_A+\frac{\partial \alpha_{CB}(.)}{\partial \bar{d}}Y_B-2t\right)=\frac{\partial \alpha_{CB}(.)}{T\partial\lambda}Y_B \tag{A10}$$

$$\frac{\partial \bar{d}}{T\partial\lambda}=\frac{\frac{\partial \alpha_{CB}(.)}{T\partial\lambda}Y_B}{\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}Y_A+\frac{\partial \alpha_{CB}(.)}{\partial \bar{d}}Y_B-2t}>0 \tag{A11}$$

Hence, Lemma 1: Everything else being equal, $\bar{d}$ is increasing in λ in other term in λT.

Sign of $\frac{\partial \bar{d}}{\partial e_A^i}$:

We have $\frac{\partial e_A^i}{\partial T}<0$, and if considering that $\frac{\partial \bar{d}}{\partial T}=\frac{-\frac{\partial \alpha_{CA}(.)}{\partial T}Y_A}{\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}Y_A-\frac{\partial \alpha_{CB}(.)}{\partial \bar{d}}Y_B-2t}<0$, we deduce that $\frac{\partial \bar{d}}{\partial e_A^i}>0$

Hence, Lemma 1: Everything else being equal, $\bar{d}$ is decreasing in e_A^i.

Sign of $\frac{\partial \bar{d}}{\partial e_B^i}$:

We have $\frac{\partial e_B^i}{T\partial\lambda}<0$, and if considering that $\frac{\partial \bar{d}}{T\partial\lambda}=\frac{\frac{\partial \alpha_{CB}(.)}{T\partial\lambda}Y_B}{\frac{\partial \alpha_{CA}(.)}{\partial \bar{d}}Y_A-\frac{\partial \alpha_{CB}(.)}{\partial \bar{d}}Y_B-2t}>0$, we deduce that $\frac{\partial \bar{d}}{\partial e_B^i}<0$.

Hence, Lemma 1: Everything else being equal, $\bar{d}$ is increasing in e_B^i.

We have just justified the last part of Lemma 1. Hence, Lemma 1: Everything else being equal, $\bar{d}$ is decreasing in T and in e_A^i and increasing in λT or λ and in e_B^i.

Proposal 1: Immediate from Lemma 1

We know that $\bar{d}_{e_A^i} = \frac{\partial \bar{d}}{\partial e_A^i} < 0$ and $\frac{\partial e_{CA}}{\partial \bar{d}} > 0$. Thus, an increase in the effort of the thief in A leads to a reduction in the distance that renders the thief in C indifferent. This outcome will lead to a reduction in the effort of the thief in C operating in A. We also know that $\bar{d}_{e_B^i} = \frac{\partial \bar{d}}{\partial e_B^i} > 0$ and $\frac{\partial e_{CB}}{\partial \bar{d}} < 0$. Thus, an increase in the effort of the thief in B leads to an increase in the distance that renders the thief in C indifferent. This outcome will then lead to a reduction in the effort of the thief in C operating in B. Hence, proposition 1.

Proposal 2: Proof

Supposing that institution B chooses an endowment $\lambda T^* < \lambda^* T^*$ creates two problems. On the one hand, it continues to attract thieves from C to its territory, and on the other hand, it increases the effort e_B^* of the thieves who are in space B. These two effects lead to an increase in the proportion of production stolen in B. Since $\lambda^* T^*$ is the optimal endowment in this circumstance, the loss due to the additional theft is much greater than the gain due to the reduction in the deterrent endowment. Endowment λT^* is thus not an optimal choice for institution B. A deviation for $\lambda T^* > \lambda^* T^*$ is desirable if

$$-\left[\sum_{i=1}^{\cdot} (\alpha_{B1}^i + \alpha_{B2}^i e_{B1}^i) + \left(\bar{d}_2 + \bar{d}_4 e_{B1}^i\right)\left(\sum_{j=1}^{\cdot} p_{CB}^j\right)\right.$$
$$\left. + (1-\bar{d})(\sum_{j=1}^{\cdot} (p_{CB1}^j + p_{CB2}^j e_{CB1}^j))\right] Y_B - 1 < 0.$$

This means that $-\left[\sum_{i=1}^{\cdot} (\alpha_{B1}^i + \alpha_{B2}^i e_{B1}^i) + \left(\bar{d}_2 + \bar{d}_4 e_{B1}^i\right)\left(\sum_{j=1}^{\cdot} p_{CB}^j\right)\right] Y_B < 0$ because $\left[(1-\bar{d})(\sum_{j=1}^{\cdot} (p_{CB1}^j + p_{CB2}^j e_{CB1}^j))\right] Y_B = 1$ with $\bar{d} = 0$.

Therefore, we have $\left[\sum_{i=1}^{\cdot} (\alpha_{B1}^i + \alpha_{B2}^i e_{B1}^i) + \left(\bar{d}_2 + \bar{d}_4 e_{B1}^i\right)\left(\sum_{j=1}^{\cdot} \alpha_{CB}^j\right)\right] Y_B \geq 0$. However, $\left[\sum_{i=1}^{\cdot} \alpha_{B2}^i e_{B1}^i\right] Y_B < 0$, which is equivalent to $\alpha_{B2}^i > 0$. Then, $\alpha_{B2}^i(\lambda^* T^*,\ e_B^*) = 0$, and $\alpha_{B2}^i(\lambda^* T^*,\ 1) \leq 0$. We can therefore write:

$$\sum_{i=1}^{\cdot} \alpha_B^{i*}(\lambda^* T^*,\ 1) + \sum_{j=1} \alpha_{CB}^{j*}(\lambda^* T^*,\ 1,1) + \alpha_{B2}^{i*}(\lambda^* T^*,\ 1)$$
$$\leq \sum_{i=1}^{\cdot} \alpha_B^{i*}(\lambda^* T^*,\ 1) + \sum_{j=1} \alpha_{CB}^{j*}(\lambda^* T^*,\ 1,1).$$

Because the condition was that all thieves should operate in B, and no thief would operate in A, e_B^{i*} being the optimum effort, we have $\sum_{i=1}^{\cdot} \alpha_B^{i*}(\lambda^* T^*,\ 1) + \sum_{j=1} \alpha_{CB}^{j*}(\lambda^* T^*,\ 1,1) < \sum_{i=1}^{\cdot} \alpha_B^{i*}(\lambda^* T^*,\ e_B^*) + \sum_{j=1} \alpha_{CB}^{j*}(\lambda^* T^*,\ 1,1)$ because the optimum effort is exceeded. Hence, condition (iii) shows that

$$\sum_{i=1}^{\cdot} \alpha_B^{i*}(\lambda^* T^*,\ 1) + \sum_{j=1} \alpha_{CB}^{j*}(\lambda^* T^*,\ 1,1) + \alpha_{B2}^{i*}(\lambda^* T^*,\ 1) \leq 0$$

In a symmetric way, so that A does not deviate from T^*, it is necessary that $\alpha_2(T^*, 0) \geq 0$.

Therefore, we have $\alpha(T^*,\ 0) + \alpha_{CA}(T^*,\ 0,0) = 0$.

We can therefore write the condition (i), $\alpha(T^*,\ 0) + \alpha_{CA}(T^*,\ 0,0) + \alpha_2(T^*,0) \geq 0$.

Let us consider the condition $\Big[(1-\bar{d})(\sum_{j=1}^{\cdot}(\alpha_{CB1}^j + \alpha_{CB2}^j e_{CB1}^j + \alpha_{CB3}^j \bar{d}_2 + \alpha_{CB3}^j \bar{d}_4 e_{B1}^i))\Big] Y_B = 1$ with $\bar{d} = 0$.

We know that $\Big[(\sum_{j=1}^{\cdot} \alpha_{CB2}^j e_{CB1}^j\Big] Y_B \leq 0$, which means that $\alpha_{CB2}^j \geq 0$.

However, when the effort is at its highest level (1), $\alpha_{CB2}^*(\lambda^* T^*,\ 1) \leq 0$.

Applying the same reasoning as in the previous case, we deduce the condition (iv), in which

$$\alpha(\lambda^* T^*,\ 1) + \alpha_{CB}(\lambda^* T^*,\ 1,1) + \alpha_{CB2}^*(\lambda^* T^*,\ 1) \leq 0.$$

Symmetric reasoning, ensuring that A does not deviate, leads to condition (ii). We know that

$$\left[(\sum_{j=1}^{\cdot} \alpha_{CA2}^j e_{CA1}^j\right] Y_A \leq 0, \text{ which means that } \alpha_{CA2}^j \geq 0.$$

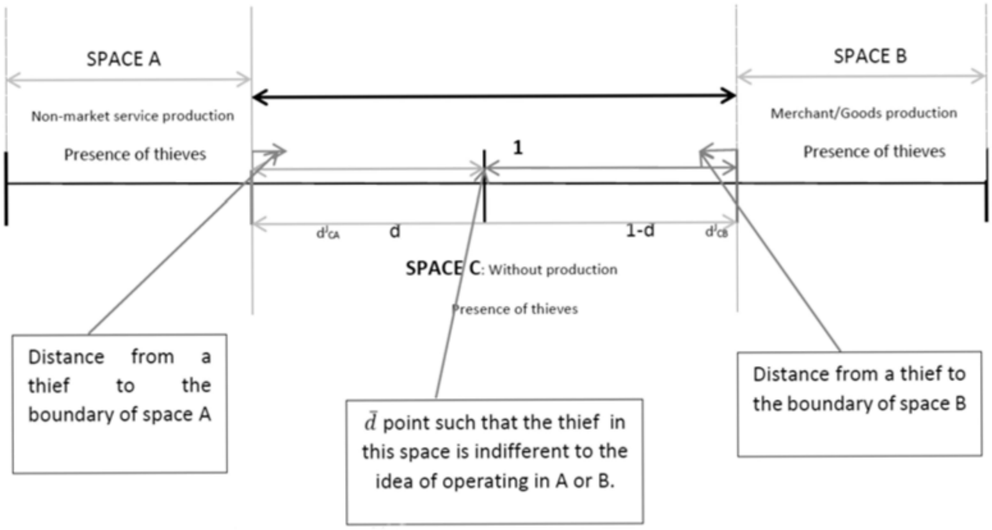
Therefore, we have $\alpha_{CA2}^*(T^*,\ 0) \geq 0$.

Finally, condition (ii) is $\alpha(T^*,\ 0) + \alpha_{CA}(T^*,\ 0,0) + \alpha_{CA2}(T^*,0,0) \geq 0$.

Condition (v) ensures that none of the institutions has an interest in deviating, that is, in taking the place of the other.

FIGURE 1

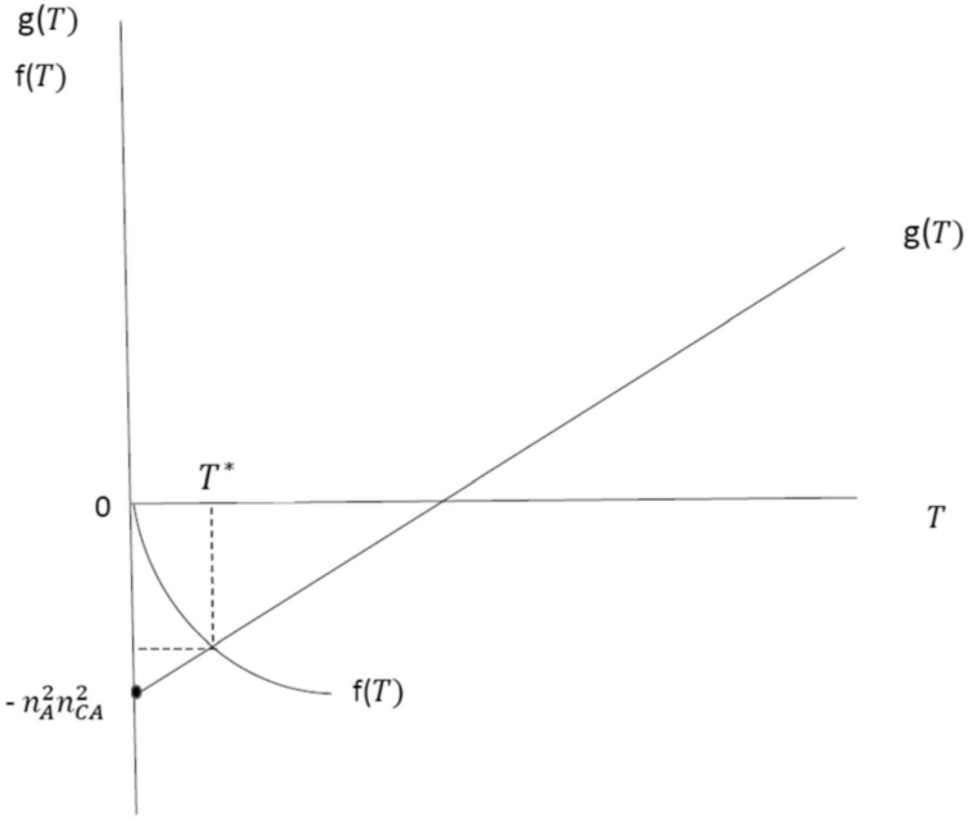
THE GEOGRAPHY OF THE MODEL



SOURCE : Author.

FIGURE 2

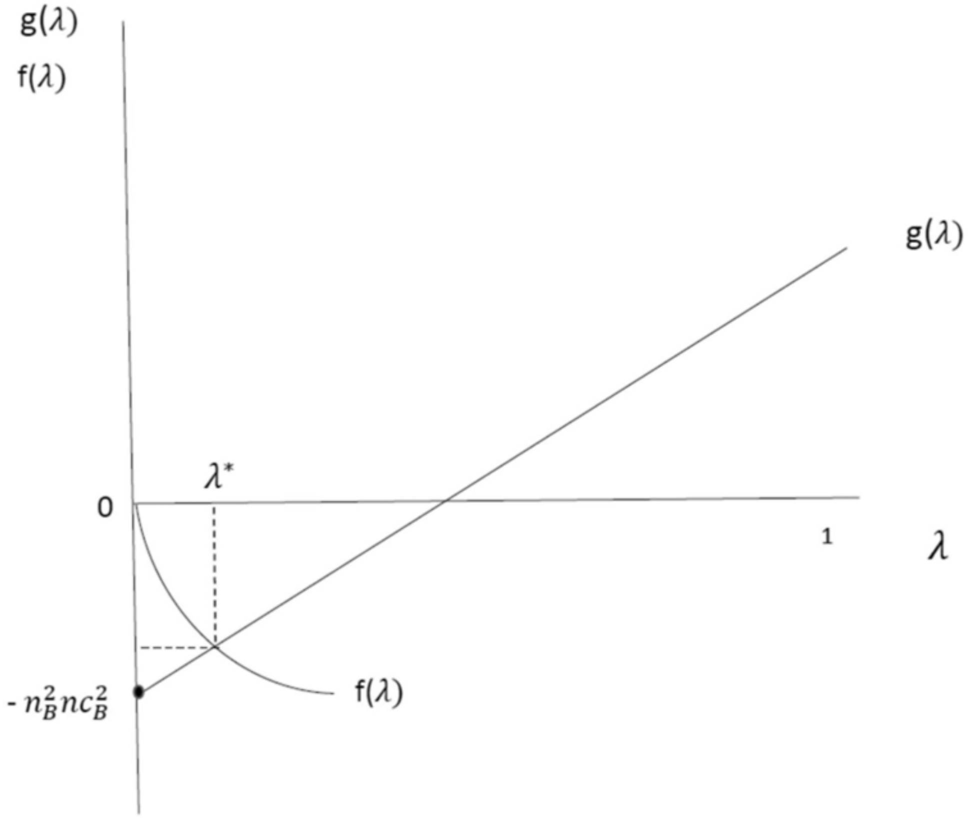
VALUE OF T^* AT EQUILIBRIUM



SOURCE : Author.

FIGURE 3

VALUE OF λ^* AT EQUILIBRIUM



SOURCE : Author.

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